Modeling and Errors

- A model is a symbolic representation of the real thing (nature).
- A model could be experimental, analytical, or numerical or some combination of the three.
- Solutions of all models are approximations, whether an approximate solution is an acceptable is a decision that is based on additional information, such as: correlation with results from other models, experience, intuition, usefulness.

Errors in FEM

1. Modeling error
2. Discretization error
3. Numerical error

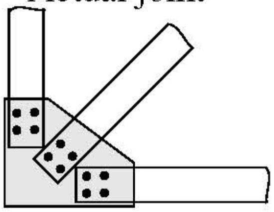
Modeling error

Error that arise from the description of the boundary value problem (BVP): Geometric description, material description, loading, boundary conditions, type of analysis.

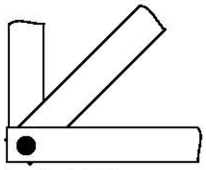
- What physical details are important in the BVP description?

  Should a mechanically fastened joint be modeled as a pin joint, welded joint, or a flexible joint.

Actual joint

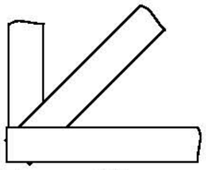


Pin joint



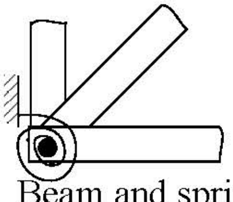
Axial Elements

Welded joint



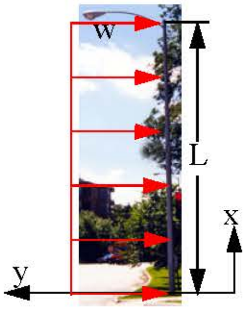
Beam Elements

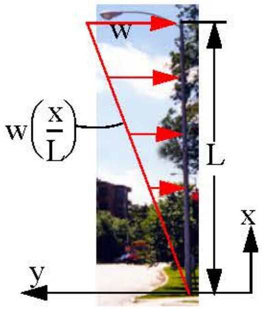
Flexible joint

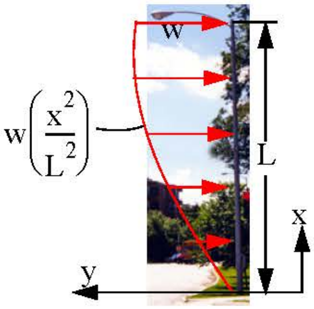


Beam and spring elements

How should the load be modeled?



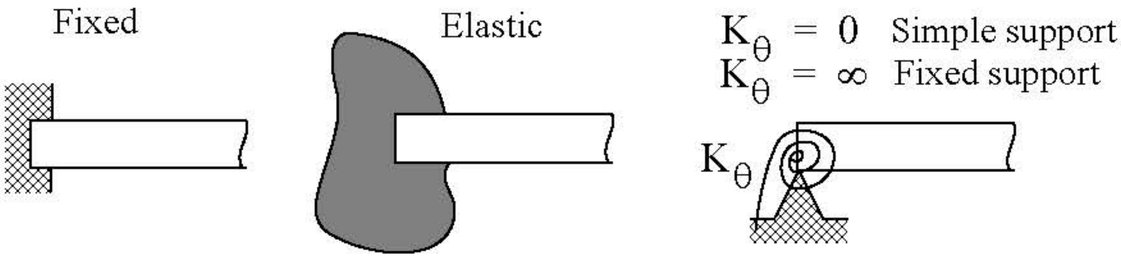




Should the properties of the adhesive be included or ignored in a bonded joint?

Should the material be modeled as isotropic or orthotropic?

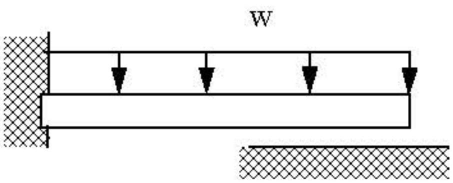
How should the support be modeled? i.e., what are the appropriate boundary conditions.

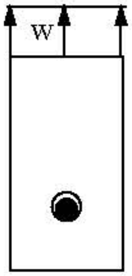


What type of analysis should be conducted?

Should you conduct a linear or non-linear analysis?

1. Material non-linearity: Stress and strain are non-linearly related.
2. Geometric non-linearity: Strain and displacement non-linearly related. (large deformation or strain)
3. Contact problem: The contact length changes with load.
    (i) No friction.
    (ii) With friction—need the slip ($F_f = \mu N$) and no slip boundary($F_f < \mu N$).





Should buckling analysis be conducted?

For time dependent problems should you conduct a dynamic or quasi-static analysis? Should the material be modeled as elastic or viscoelastic?

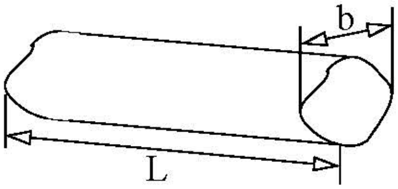
Discretization errors

- Errors that arises from creation of the mesh.

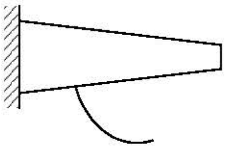
Elements in FEM are based on analytical models. All assumptions that are made in the analytical models are applicable to FEM elements.

- What type of elements should be used?

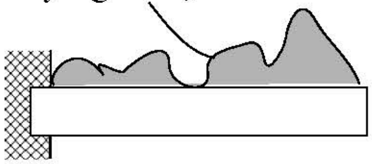
Should 1-d element be used?



L » b—1-d OK

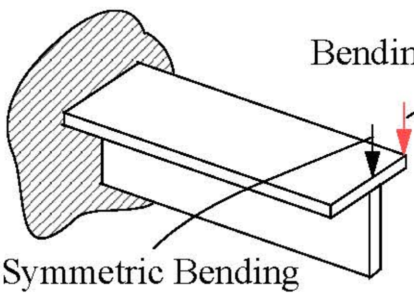


Very steep taper, 1-d Not OK

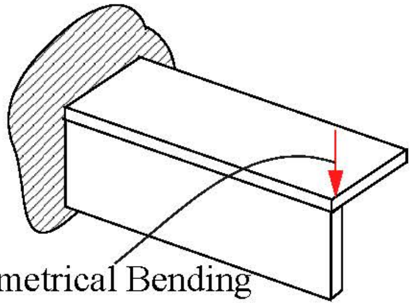


Rapidly varying load, 1-d Not OK

Should beam element, which is based on symmetric bending, be used?



Symmetric Bending



Unsymmetrical Bending

- What type of 2-d (plane stress, plane strain) or 3-d element should you use.?
- What mesh density should you use?

Too fine a mesh results in large computer time that may prevent optimization or parametric studies or non-linear analysis. Too coarse a mesh may result in high inaccuracies. Start with a coarse mesh, study the results and then refine the mesh as needed.

- How accurately should the geometry be modelled?

Errors from modeling of geometric are generally small. For the same computational effort higher returns in accuracy are obtained in better modelling of displacement--Isoparametric elements are adequate.

Numerical Errors

Errors that arise from finite digit arithmetic and use of numerical methods.

- Integration error

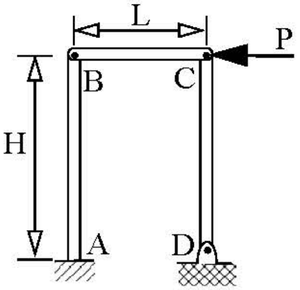
  Few Gauss points leads to numerical instabilities. Large number of Gauss points are computationally expensive and may result in overly stiff elements leading to higher errors.

- Round off error

  The finite digit arithmetic causes these errors, but the growth of round off errors are dictated by several factors. Need to avoid: adding or subtracting very large and very small numbers; dividing by small numbers.

  (i) The manner in which algorithms are written in the computer codes. Non-dimensionalizing the problem will always help.

  (ii) Large differences in physical dimensions.



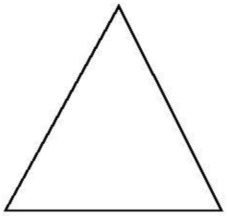
$\frac{H}{L} \to \infty$ Large $\quad K_{BC} \gg K_{AB} \quad K_{BC} \gg K_{CD}$

Can BC be modeled as rigid?

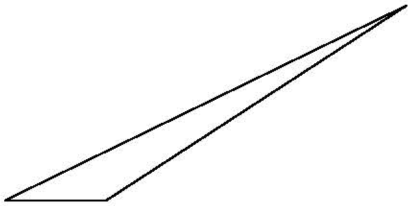
(iii) Large differences in stiffness caused my large differences in material properties (or dimensions).

Can steel be modeled as rigid?

(iv) Elements with poor aspect ratio: ratio of largest to smallest dimension in an element.



Good Aspect Ratio



Poor Aspect Ratio

- Matrix conditioning error

  The coefficients in the matrix of the algebraic equations are affected by the above described numerical errors and it is in the solution of the algebraic equations one realizes something has gone wrong.

Example of poor conditioning:

$$\begin{bmatrix} 1 & -1 \\ -1 & 1.001 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 2 \\ -3 \end{Bmatrix} \qquad \Rightarrow \qquad \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} -998 \\ -1000 \end{Bmatrix}$$

$$\begin{bmatrix} 1 & -0.999 \\ -0.999 & 1.001 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 2 \\ -3 \end{Bmatrix} \qquad \Rightarrow \qquad \begin{Bmatrix} u_1 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} -331.777 \\ -334.11 \end{Bmatrix}$$

- Small changes produce large swings in the solution. Thus, a solution may not be correct.
- Matrix conditioning is a measure of diagonal dominance in a matrix.
- Poorly conditioned matrices have a determinant that tends towards zero.

Mesh Refinement

- Elements with high strain energy identify the region of the body where mesh should be refined.

  The *h-method* of mesh refinement reduces the size of element.

  The *p-method* of mesh refinement increases the order of polynomial in an element.

  The *r-method* of mesh refinement relocates the position of a node.

  Combinations: hr-method, hp-method, hpr-method

- Generally speaking: the p-method of mesh refinement works well for regions where stresses vary slowly; the hr- method works better suited for regions of large stress gradients.

Conclusions

1. There are many reasons for a FEM program to give errors and not work.
2. There are many reasons for a FEM program to work but give wrong results.
3. Don’t blame the software, it is your responsibility to ascertain if you have the right results or not.