EXAM 1

1. Suppose we have the following differential equation $x\left(\frac{d^2u}{dx^2}\right) + \frac{du}{dx} = 3 \qquad 0 \le x \le 1$

   and we are using the approximation of $u(x) = c_1x^2 + c_2x^3$. Set up the equations in matrix form for solving c_1 and c_2 by (a) Least square method. (b) By Galerkin's method. (YOU DO NOT NEED TO SOLVE THE EQUATIONS).

2.

(a)-(d) The order of the highest derivative on u in a functional is r. For parts (a) through (d) circle the correct answers.

(a) The differential equation will be of order r-1, r, r+1, 2r-1, 2r, 2r+1

(b) The essential boundary conditions will be on u and its detivatives up to order r-1, r, r+1, 2r-1, 2r, 2r+1

(c) The natural boundary conditions will be on the derivatives up to orderr-1, r, r+1, 2r-1, 2r, 2r+1

(d) In Rayleigh-Ritz's method the approximating function must be differentiable up to order r-1, r, r+1, 2r-1, 2r, 2r+1

(e) The set approximating functions must be independent. Circle the set(s) that are NOT independent.

set 1: $(x - x^2), (x^2 - x^3), (x^3 - x^4), (x^4 - x^5)$

set 2: $(x - x^2), (x^2 - x^3), (x^4 - x^5), (x^5 - x^6)$

set 3: $x(1 - x), x^2(1 - x), x(1 - x)^3, x(1 - x)^4$

set 4: $x(1 - x), x^2(1 - x), x(1 - x)^2, x(1 - x)^3$

(f) The set approximating functions must be complete. Circle the set(s) that are NOT complete.

set 1: $(x - x^2), (x^2 - x^3), (x^3 - x^4), (x^4 - x^5)$

set 2: $(x - x^2), (x^2 - x^3), (x^4 - x^5), (x^5 - x^6)$

set 3: $x(1 - x), x^2(1 - x), x(1 - x)^3, x(1 - x)^4$

set 4: $x(1-x), x^2(1-x), x(1-x)^2, x(1-x)^3$

(g)- (h) B(u,v) is a bilinear functional. For parts (g) and (h) circle the correct answers.
(g) B(2u,3v) will equal to 2B(u,v), 3B(u,v), 5B(u,v), 6B(u,v)

(h) $B(2u_1+3u_2,v)$ will equal to $2B(u_1,v)+3B(u_2,v)$, $3B(u_1,v)+2B(u_2,v)$, $5[B(u_1,v)+B(u_2,v)]$, $6[B(u_1,v)+B(u_2,*v)]$

(i) u_1, u_2, and u_3 are all independent variables and the condition $a_1u_1 + a_2u_2 + b_1u_1 + b_2(u_2 + u_3) + b_3u_3 = 0$ is true. Write all the conditions on the contants a's and b's.

(j)-(k)In Rayleigh-Ritz we approximate as: $u = \phi_o + \sum_{i=1}^{N} c_i\phi_i$. Write the functions ϕ_o and ϕ_1 for one parameter solution for parts(j) and (k)

(j) Essential boundary conditions are: $u(0) = 4$ $u(1) = 0$

$\phi_o =$ $\phi_1 =$

(k) Essential boundary condition is: $u(0) = 0$ and natural boundary condition is $\frac{du}{dx}(1) = 2$

$\phi_o =$ $\phi_1 =$

3. A functional is given below. By finding the stationary value of I, obtain the following: (a) The differential equation. (b) *All possible* essential boundary conditions. (c) *All possible* natural boundary conditions.

$$I = \int_0^1 \left[(1+x)^2\left(\frac{du}{dx}\right)^2 + 3u^2 - x^2u\right]dx$$