Deflection of Symmetric Beams

Learning objective

- Learn to formulate and solve the boundary-value problem for the deflection of a beam at any point.

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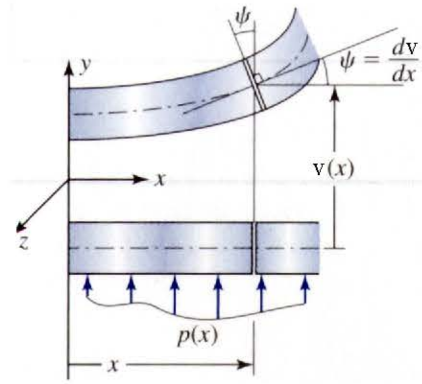
Second-Order Boundary Value Problem

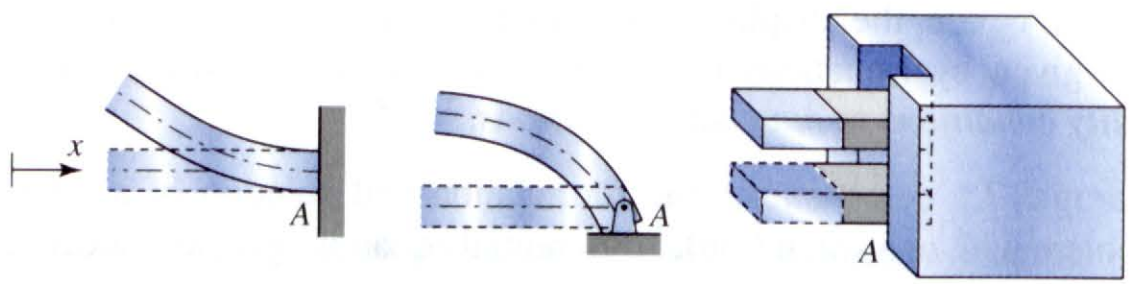


- The deflected curve represented by v(x) is called the Elastic Curve.

Differential equation: $\boldsymbol{M}_z = EI_{zz}\frac{d^2 v}{dx^2}$

- The mathematical statement listing all the differential equations and all the conditions necessary for solving for v(x) is called the Boundary Value Problem for the beam deflection.

Boundary Conditions



$$v(x_A) = 0 \qquad v(x_A) = 0 \qquad \frac{dv}{dx}(x_A) = 0$$

$$\frac{dv}{dx}(x_A) = 0$$

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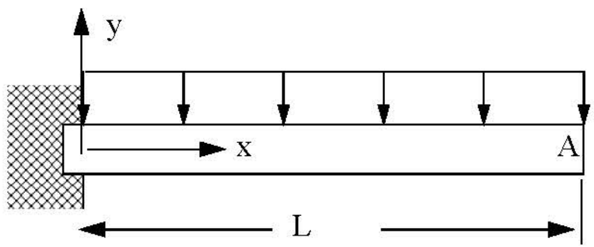
C7.1 In terms of w, P, L, E, and I determine (a) equation of the elastic curve. (b) the deflection of the beam at point A.



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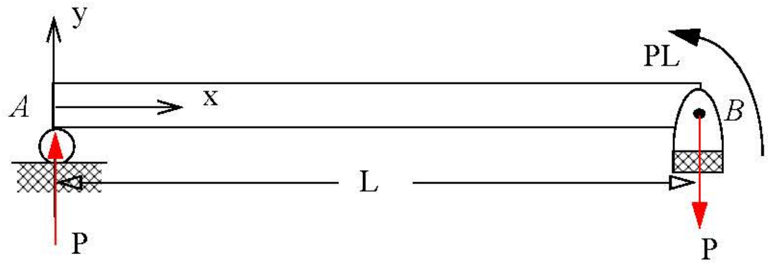
Class Problem 7.1

Write the boundary value problem to determine the elastic curve. Note the reaction force at the support has been calculated for you.



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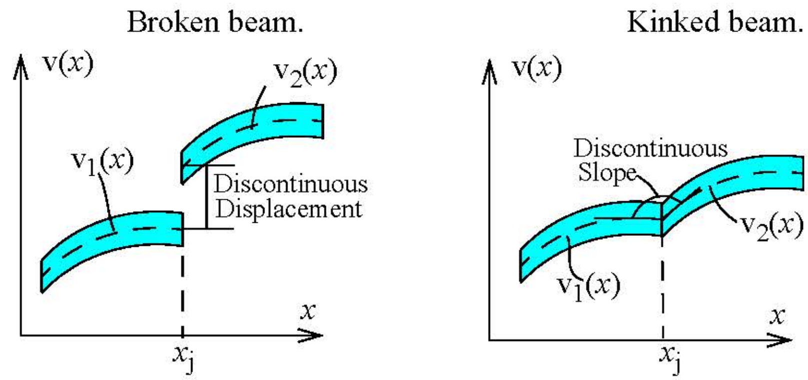
Continuity Conditions

- The internal moment M_z will change with change in applied loading.
- Each change in M_z represents a new differential equation, hence new integration constants.



$$v_1(x_j) = v_2(x_j)$$

$$\frac{dv_1}{dx}(x_j) = \frac{dv_2}{dx}(x_j)$$

- 'continuity conditions', also known as 'compatibility conditions' or 'matching conditions'.

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C7.2 In terms of w, L, E, and I, determine (a) the equation of the elastic curve. (b) the deflection at x = L.

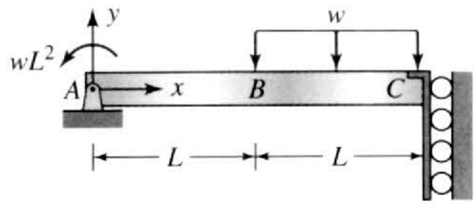

Fig. C1.2

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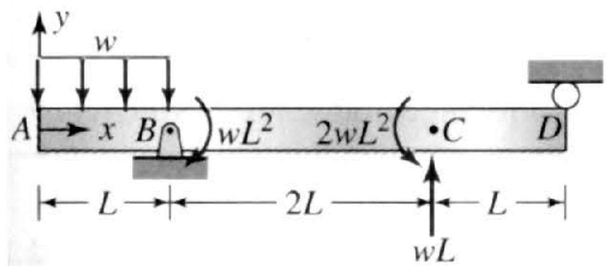
Class Problem 7.2

Write the boundary value problem for determining the deflection of the beam at any point *x*. Assume EI is constant. Do not integrate or solve.



The internal moments are:

In *AB*: $M_1 = -\dfrac{wx^2}{2}$ In *BC*: $M_2 = \dfrac{wLx}{6} + \dfrac{wL^2}{3}$

In *CD*: $M_3 = \dfrac{7wLx}{6} - \dfrac{14wL^2}{3}$

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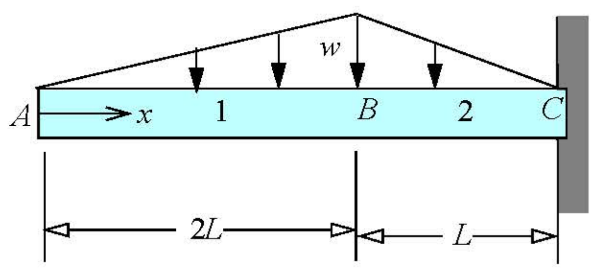
Class Problem 7.3

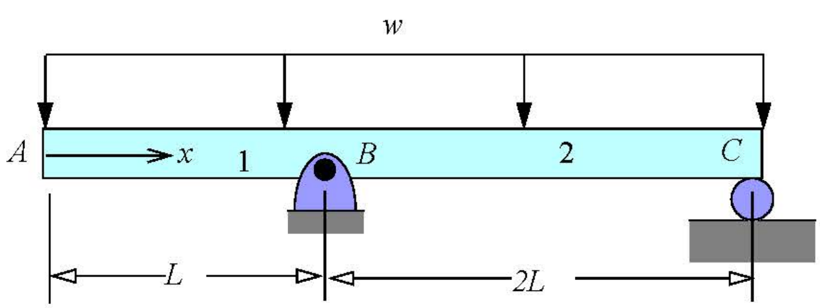
v_1 and v_2 represents the deflection in segment *AB* and *BC*. For the beams shown, identify all the conditions from the table needed to solve for the deflection v(x) at any point on the beam.

(a) $v_1(0) = 0$	(e) $v_2(2L) = 0$	(i) $v_1(L) = v_2(L)$
(b) $v_1(L) = 0$	(f) $v_2(3L) = 0$	(j) $v_1(2L) = v_2(2L)$
(c) $v_2(L) = 0$	(g) $\frac{dv_1}{dx}(0) = 0$	(k) $\frac{dv_1}{dx}(L) = \frac{dv_2}{dx}(L)$
(d) $v_1(2L) = 0$	(h) $\frac{dv_2}{dx}(3L) = 0$	(l) $\frac{dv_1}{dx}(2L) = \frac{dv_2}{dx}(2L)$

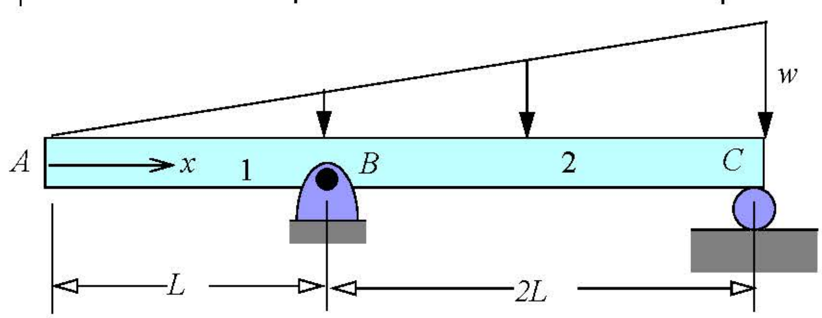
Beam 1



Beam 2



Beam 3



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