Axial Members

- Members with length significantly greater than the largest cross-sectional dimension and with loads applied along the longitudinal axis.

Cables of Mackinaw bridge

(a)

Hydraulic cylinders in a dump truck

(b)

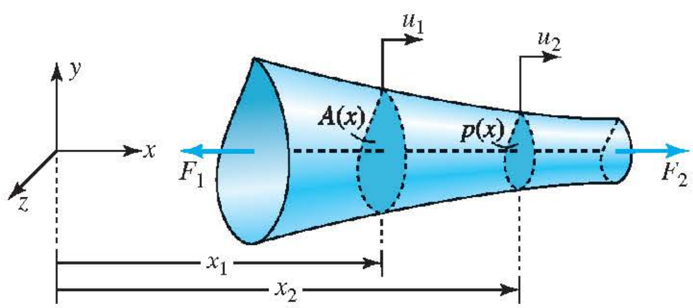
Learning objectives are:

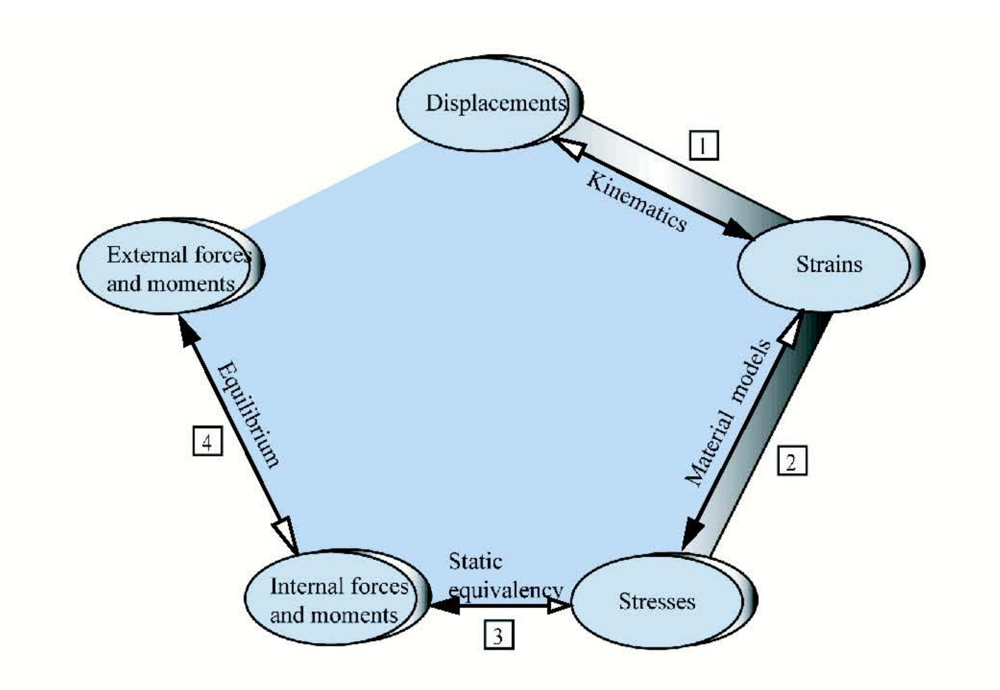
- Understand the theory, its limitations, and its applications for design and analysis of axial members.
- Develop the discipline to draw free body diagrams and approximate deformed shapes in the design and analysis of structures.

Theory

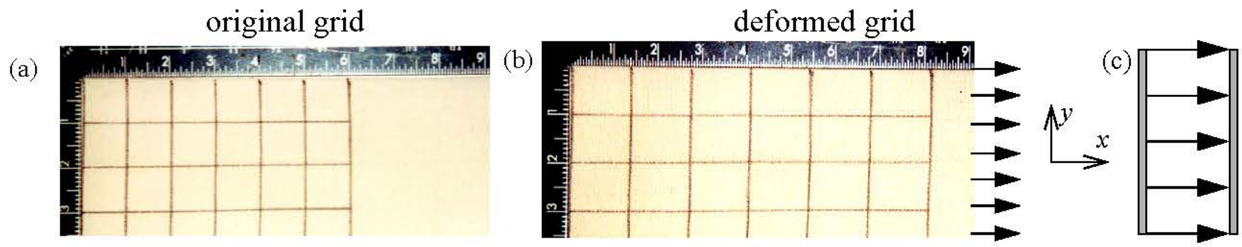
Theory Objective

- to obtain a formula for the relative displacements (u_2-u_1) in terms of the internal axial force *N*.
- to obtain a formula for the axial stress σ_{xx} in terms of the internal axial force *N*.





Kinematics



Assumption 1 Plane sections remain plane and parallel. $u = u(x)$

- The displacement u is considered positive in the positive x-direction.

Assumption 2 Strains are small. $\varepsilon_{xx} = \dfrac{du(x)}{dx}$

Material Model

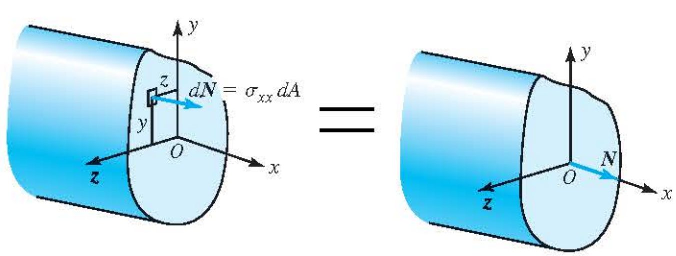
Assumption 3 Material is isotropic.

Assumption 4 Material is linearly elastic.

Assumption 5 There are no inelastic strains.

From Hooke's Law: $\sigma_{xx} = E\varepsilon_{xx}$, we obtain $\sigma_{xx} = E\dfrac{du}{dx}$

Internal Axial Force



$$N = \int_A \sigma_{xx} dA$$

$$N = \int_A E\frac{du}{dx} dA = \frac{du}{dx}\int_A E dA$$

- For pure axial problems the internal moments (bending) M_y and M_z must be zero.
- For homogenous materials all external and internal axial forces must pass through the centroids of the cross-section and all centroids must lie on a straight line.

Axial Formulas

Assumption 6 Material is homogenous across the cross-section.

$$N = E\frac{du}{dx}\int_A dA = EA\frac{du}{dx} \qquad or \qquad \frac{du}{dx} = \frac{N}{EA}$$

$$\sigma_{xx} = E\frac{du}{dx} = E\left(\frac{N}{EA}\right) \qquad or \qquad \sigma_{xx} = \frac{N}{A}$$

- The quantity EA is called the Axial rigidity.

Assumption 7 Material is homogenous between x_1 and x_2.

Assumption 8 The bar is not tapered between x_1 and x_2.

Assumption 9 The external (hence internal) axial force does not change with x between x_1 and x_2.

$$u_2 - u_1 = \frac{N(x_2 - x_1)}{EA}$$

Two options for determining internal axial force ***N***

- ***N*** is always drawn in tension at the imaginary cut on the free body diagram.
  - Positive value of σ_{xx} will be tension.
  - Positive u_2-u_1 is extension.
  - Positive u is in the positive x-direction.
- ***N*** is drawn at the imaginary cut in a direction to equilibrate the external forces on the free body diagram.
  - Tension or compression for σ_{xx} has to be determined by inspection.
  - Extension or contraction for δ=u_2-u_1 has to be determined by inspection.
  - Direction of displacement u has to be determined by inspection.

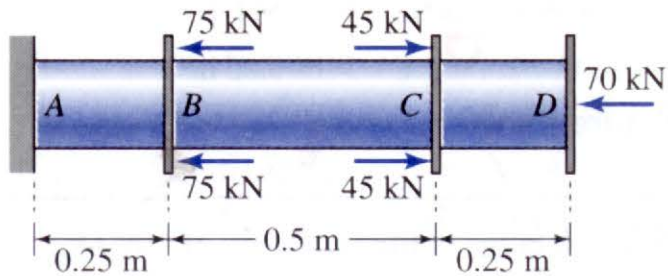
Axial stresses and strains

- all stress components except σ_{xx} can be assumed zero.

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E}$$

$$\varepsilon_{yy} = -\left(\frac{\nu\sigma_{xx}}{E}\right) = -\nu\varepsilon_{xx} \qquad \varepsilon_{zz} = -\left(\frac{\nu\sigma_{xx}}{E}\right) = -\nu\varepsilon_{xx}$$

C4.1 Determine the internal axial forces in segments AB, BC, and CD by making imaginary cuts and drawing free body diagrams.



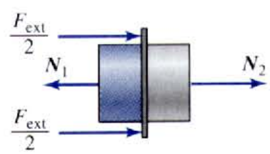
Axial Force Diagrams

- An axial force diagram is a plot of internal axial force ***N*** vs. x
- Internal axial force jumps by the value of the external force as one crosses the external force from left to right.
- An axial template is used to determine the direction of the jump in ***N***.
- A template is a free body diagram of a small segment of an axial bar created by making an imaginary cut just before and just after the section where the external force is applied.

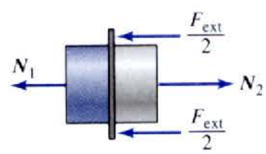
Template 1 Equation

$$N_2 = N_1 - F_{ext}$$

Template 1



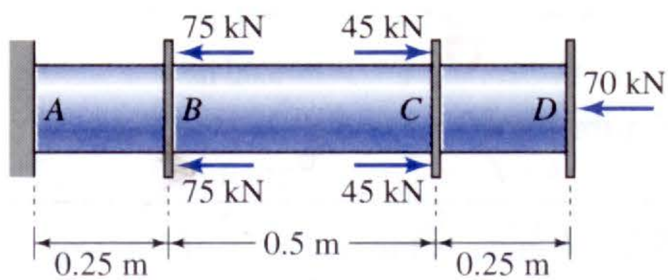
Template 2



Template 2 Equation

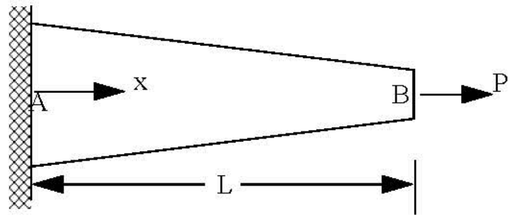
$$N_2 = N_1 + F_{ext}$$

C4.2 Determine the internal axial forces in segments AB, BC, and CD by drawing axial force diagram.



C4.3 The axial rigidity of the bar in problem 4.8 is EA = 80,000 kN. Determine the movement of section at C.

C4.4 The tapered bar shown in Fig. C4.4 has a cross-sectional area that varies with x as given. Determine the elongation of the bar in terms of P, L, E and K.



$$A = K(4L - 3x)$$

Fig. C4.4

C4.5 The columns shown has a length L, modulus of elasticity E, specific weight γ, and length a as the side of an equilateral triangle. Determine the contraction of the column in terms of L, E, γ, and a.

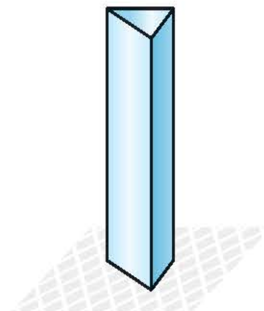


Fig. C4.5

C4.6 A hitch for an automobile is to be designed for pulling a maximum load of 3,600 lbs. A solid-square-bar fits into a square-tube, and is held in place by a pin as shown. The allowable axial stress in the bar is 6 ksi, the allowable shear stress in the pin is 10 ksi, and the allowable axial stress in the steel tube is 12 ksi. To the nearest 1/16th of an inch, determine the minimum cross-sectional dimensions of the pin, the bar and the tube. Neglect stress concentration. (Note: Pin is in double shear)

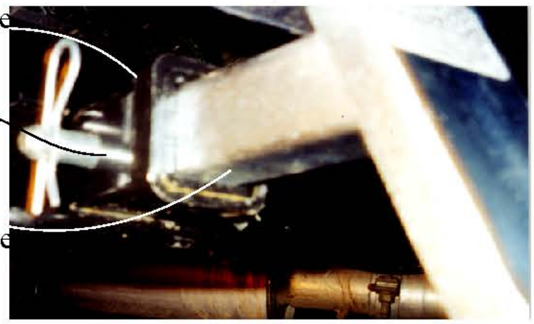


Fig. C4.6

Structural analysis

$$\delta = \frac{NL}{EA}$$

- δ is the deformation of the bar in the undeformed direction.
- If N is a tensile force then δ is elongation.
- If N is a compressive force then δ is contraction.
- Deformation of a member shown in the drawing of approximate deformed geometry must be consistent with the internal force in the member that is shown on the free body diagram.
- In statically indeterminate structures number of unknowns exceed the number of static equilibrium equations. The extra equations needed to solve the problem are relationships between deformations obtained from the deformed geometry.
- Force method----Internal forces or reaction forces are unknowns.
- Displacement method---Displacements of points are unknowns.

General Procedure for analysis of indeterminate structures.

- If there is a gap, assume it will close at equilibrium.
- Draw Free Body Diagrams, write equilibrium equations.
- Draw an exaggerated approximate deformed shape. Write compatibility equations.
- Write internal forces in terms of deformations for each member.
- Solve equations.
- Check if the assumption of gap closure is correct.

C4.7 A force F= 20 kN is applied to the roller that slides inside a slot. Both bars have an area of cross-section of A = 100 mm^2 and a Modulus of Elasticity E = 200 GPa. Bar AP and BP have lengths of L_{AP}= 200 mm and L_{BP}= 250 mm respectively. Determine the displacement of the roller and axial stress in bar A.

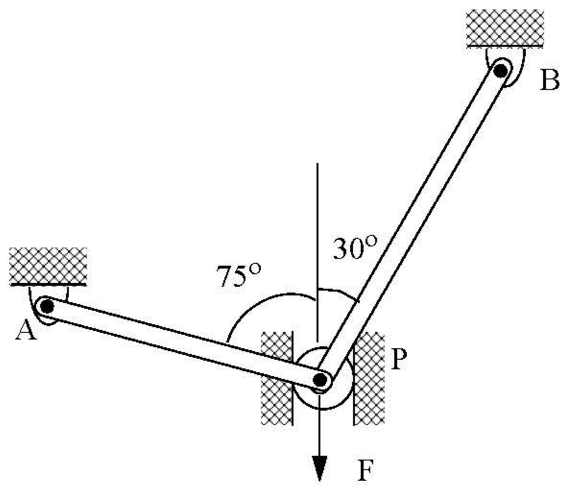


Fig. C4.7

C4.8 In Fig. C4.8, a gap exists between the rigid bar and rod A before the force F=75 kN is applied. The rigid bar is hinged at point C. The lengths of bar A and B are 1 m and 1.5 m respectively and the diameters are 50 mm and 30 mm respectively. The bars are made of steel with a modulus of elasticity E = 200 GPa and Poisson's ratio is 0.28. Determine (a) the deformation of the two bars. (b) the change in the diameters of the two bars.

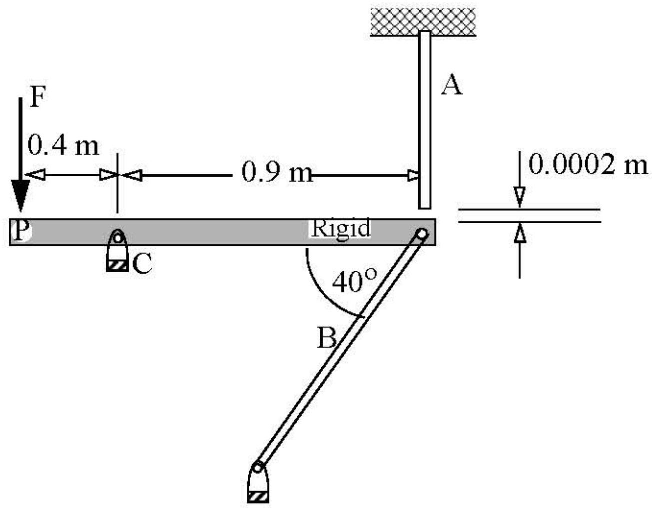


Fig. C4.8

Class Problem 4.1

Write equilibrium equations, compatibility equations, and $\delta = \frac{NL}{EA}$ for each member using the given data. **No need to solve**.

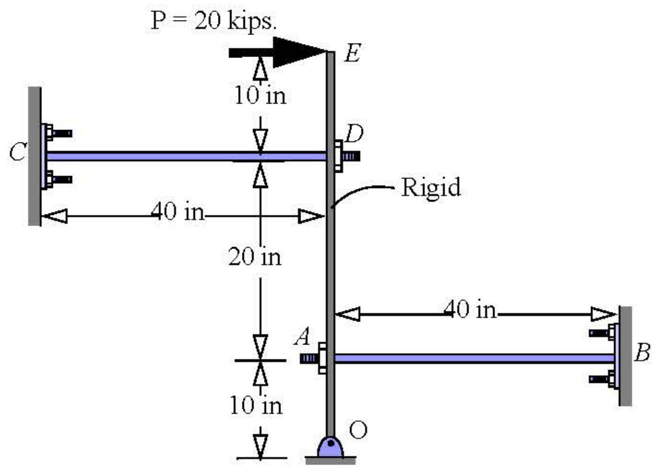
Use displacement of point E δ_E as unknown.

E = 10,000 ksi

A = 5 in^2.

$$EA = 50,000$$

$$L/EA = 0.8(10^{-3})$$



Class Problem 4.2

Write equilibrium equations, compatibility equations, and $\delta = \frac{NL}{EA}$ for each member using the given data. **No need to solve**.
Use reaction force at A (R_A) as unknown.

E = 10,000 ksi A = 5 in^2.

$EA = 50,000$