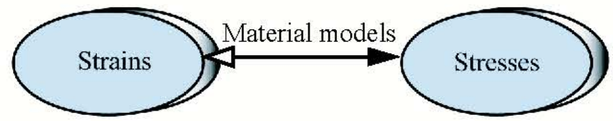
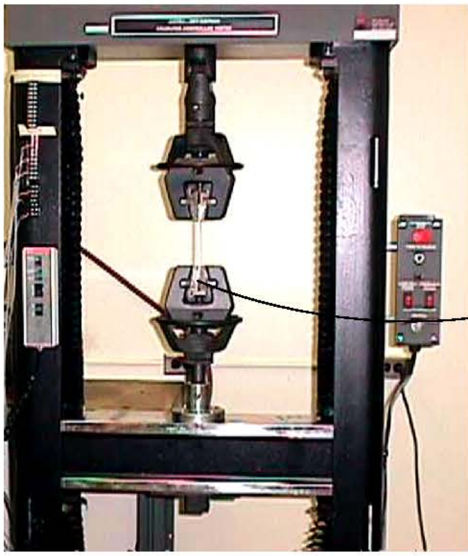
Mechanical Properties of Materials

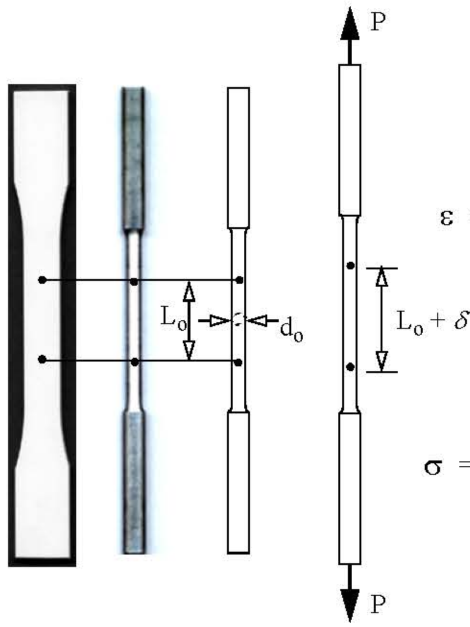


Learning objectives

- Understand the qualitative and quantitative description of mechanical properties of materials.
- Learn the logic of relating deformation to external forces.

Tension Test





$$\varepsilon = \frac{\delta}{L_o}$$

$$\sigma = \frac{P}{A_o} = \frac{P}{\pi d_o^2/4}$$

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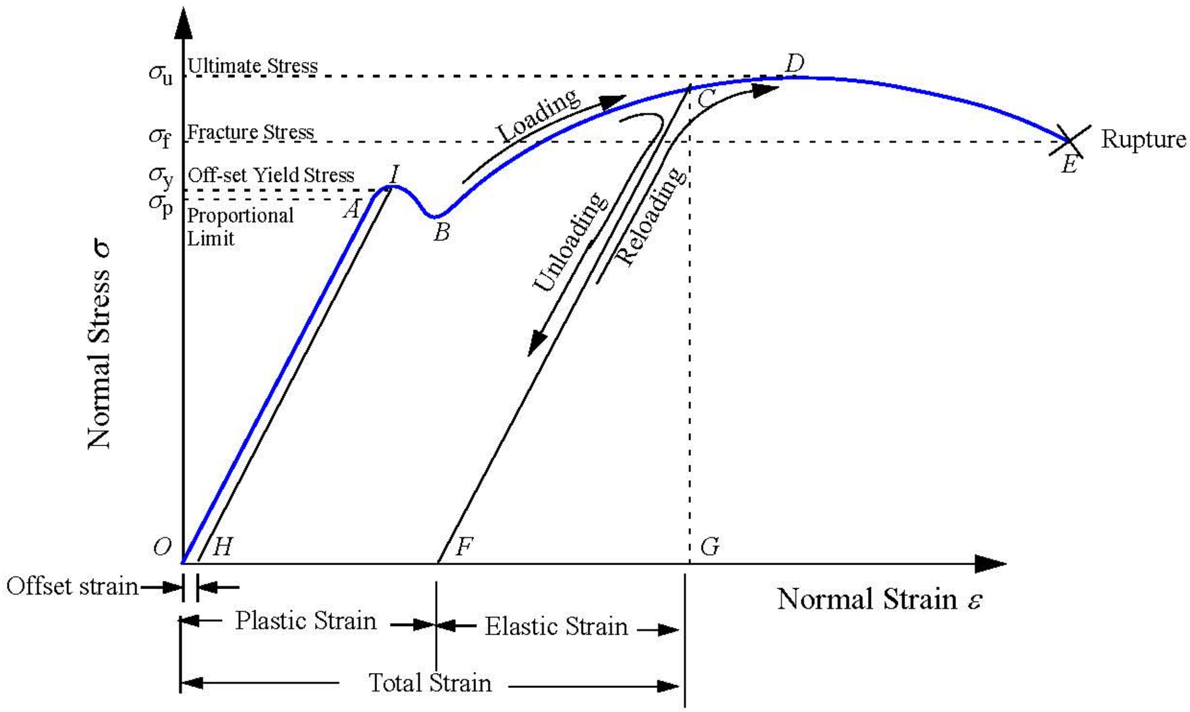
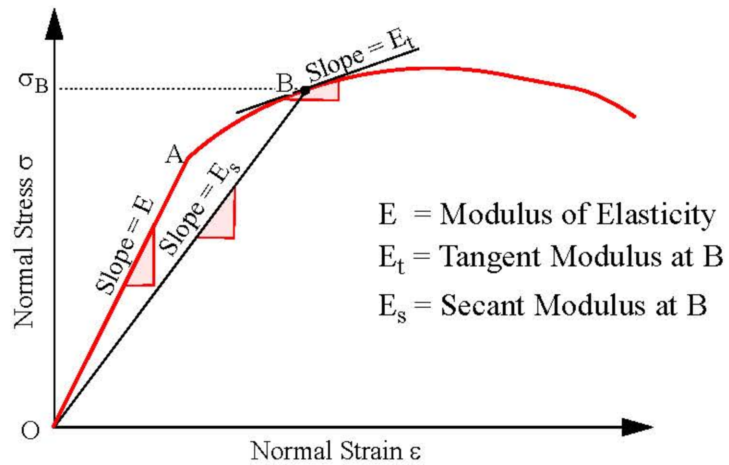


ıre 3.1 Stress–strain curve.

Definitions

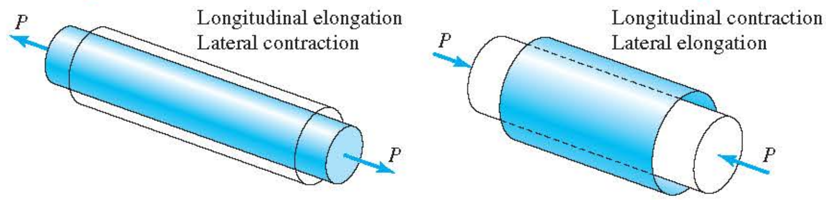
- The point up to which the stress and strain are linearly related is called the proportional limit.
- The largest stress in the stress strain curve is called the ultimate stress.
- The stress at the point of rupture is called the fracture or rupture stress.
- The region of the stress-strain curve in which the material returns to the undeformed state when applied forces are removed is called the elastic region.
- The region in which the material deforms permanently is called the plastic region.
- The point demarcating the elastic from the plastic region is called the yield point. The stress at yield point is called the yield stress.
- The permanent strain when stresses are zero is called the plastic strain.
- The off-set yield stress is a stress that would produce a plastic strain corresponding to the specified off-set strain.
- A material that can undergo large plastic deformation before fracture is called a ductile material.
- A material that exhibits little or no plastic deformation at failure is called a brittle material.
- Hardness is the resistance to indentation.
- The raising of the yield point with increasing strain is called strain hardening.
- The sudden decrease in the area of cross-section after ultimate stress is called necking.

Material Constants

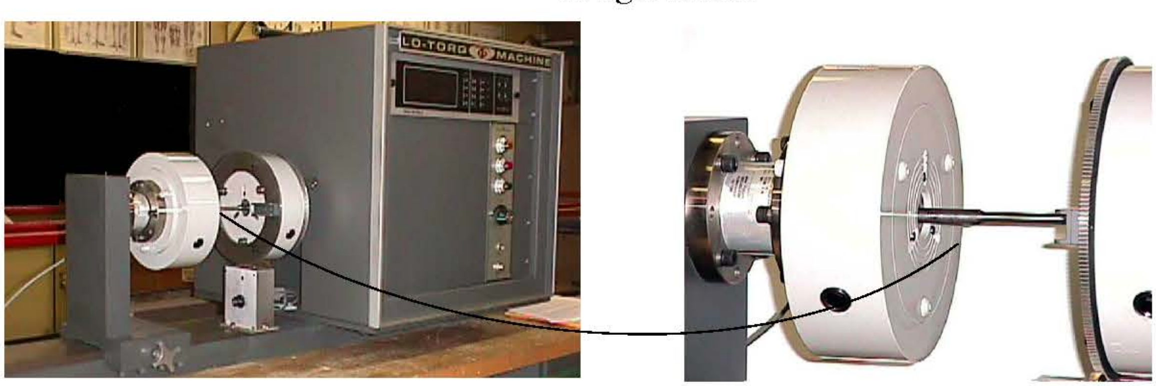


$$\sigma = E\varepsilon$$ ----------------Hooke's Law

- E Young's Modulus or Modulus of Elasticity



- Poisson's ratio: $$\nu = -\left(\frac{\varepsilon_{lateral}}{\varepsilon_{longitudnal}}\right)$$

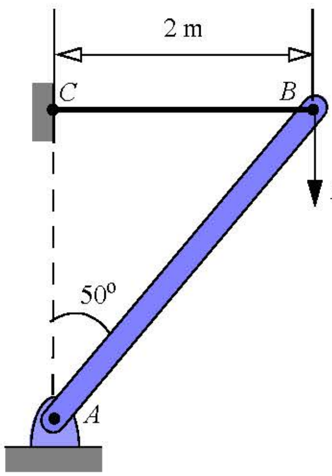


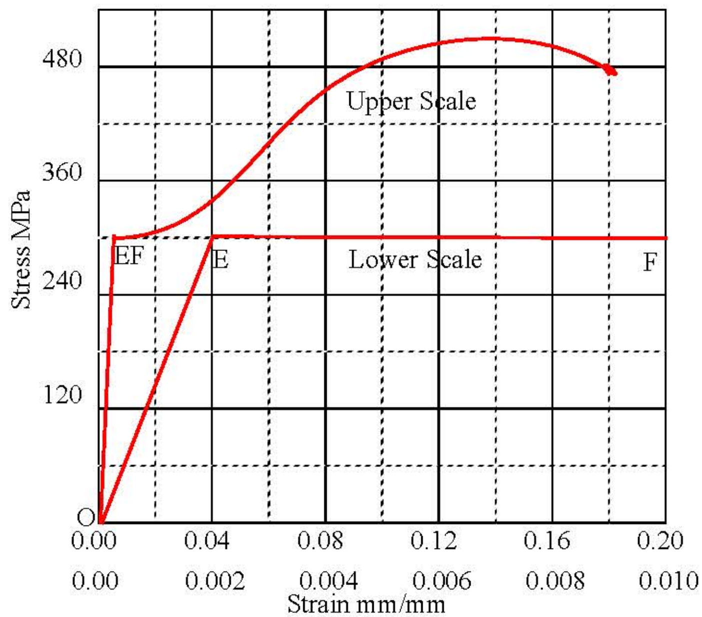
$$\tau = G\gamma$$

G is called the Shear Modulus of Elasticity or the Modulus of Rigidity

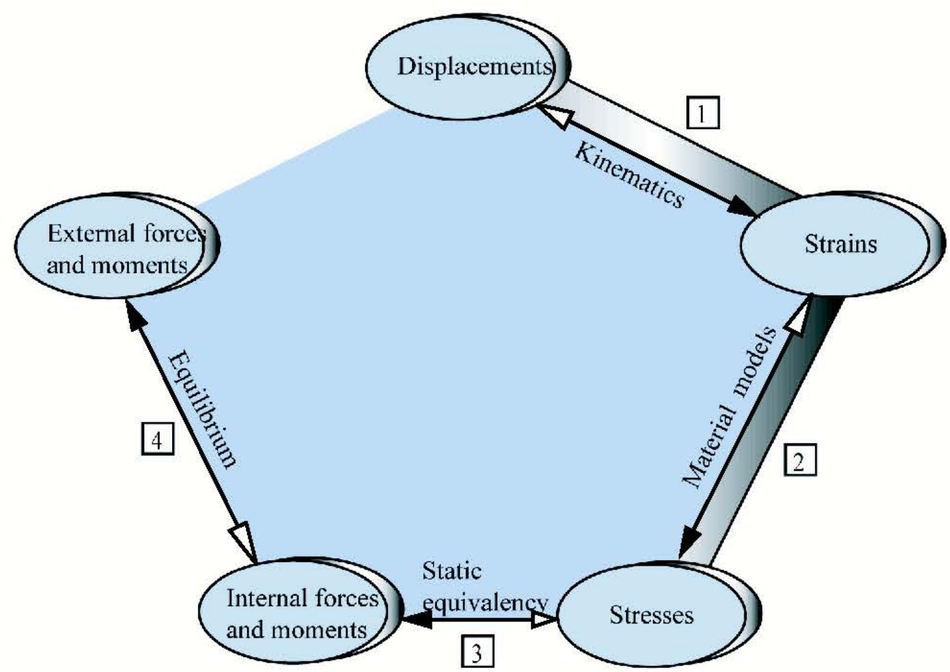
C3.1 An aluminum rectangular bar has a cross-section of 25 mm x 50 mm and a length of 500 mm. The Modulus of Elasticity of E = 70 GPa and a Poisson's ratio of $\nu = 0.25$. Determine the percentage change in the volume of the bar when an axial force of 300 kN is applied to the bar.

C3.2 A rigid bar *AB* of negligible weight is supported by cable of diameter 6 mm, as shown. The cable is made from a material that has a stress- strain curve shown. (a) Determine the extension of the cable when P = 10 kN. (b) What is the permanent deformation in *BC* when the load P is removed?





Logic in structural analysis



C3.3 A roller slides in a slot by the amount $\delta_P = 0.25$ mm in the direction of the force F. Both bars have an area of cross-section of $A = 100\ mm^2$ and a Modulus of Elasticity E = 200 GPa. Bar AP and BP have lengths of L_{AP}= 200 mm and L_{BP}= 250 mm respectively. Determine the applied force F.

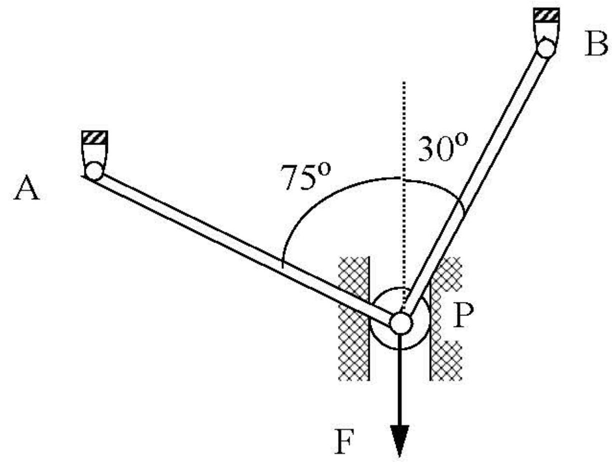


Fig. C3.3

Failure and factor of safety

- Failure implies that a component or a structure does not perform the function it was designed for.

$$K_{safety} = \frac{Failure\ producing\ value}{Computed(allowable)value} \qquad \mathbf{3.1}$$

C3.4 A gap of 0.004 in. exists between a rigid bar and bar A before a force F is applied (Figure P3.4). The rigid bar is hinged at point C. Due to force F the strain in bar A was found to be $-500\ \mu$in/in. The lengths of bars A and B are 30 in. and 50 in., respectively. Both bars have cross-sectional areas $A = 1$ in.^{2}, a modulus of elasticity $E = 30{,}000$ ksi, and yield stress of 35 ksi. Determine (a) the applied force F. (b) the factor of safety if yielding is to be avoided.

Fig. C3.4

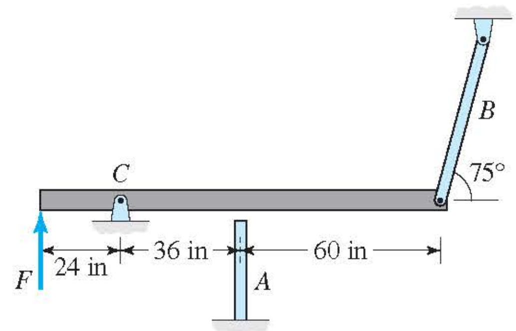


Fig. C3.4

Isotropy and Homogeneity

Linear relationship between stress and strain components:

$$\begin{aligned}
\varepsilon_{xx} &= C_{11}\sigma_{xx} + C_{12}\sigma_{yy} + C_{13}\sigma_{zz} + C_{14}\tau_{yz} + C_{15}\tau_{zx} + C_{16}\tau_{xy} \\
\varepsilon_{yy} &= C_{21}\sigma_{xx} + C_{22}\sigma_{yy} + C_{23}\sigma_{zz} + C_{24}\tau_{yz} + C_{25}\tau_{zx} + C_{26}\tau_{xy} \\
\varepsilon_{zz} &= C_{31}\sigma_{xx} + C_{32}\sigma_{yy} + C_{33}\sigma_{zz} + C_{34}\tau_{yz} + C_{35}\tau_{zx} + C_{36}\tau_{xy} \\
\gamma_{yz} &= C_{41}\sigma_{xx} + C_{42}\sigma_{yy} + C_{43}\sigma_{zz} + C_{44}\tau_{yz} + C_{45}\tau_{zx} + C_{46}\tau_{xy} \\
\gamma_{zx} &= C_{51}\sigma_{xx} + C_{52}\sigma_{yy} + C_{53}\sigma_{zz} + C_{54}\tau_{yz} + C_{55}\tau_{zx} + C_{56}\tau_{xy} \\
\gamma_{xy} &= C_{61}\sigma_{xx} + C_{62}\sigma_{yy} + C_{63}\sigma_{zz} + C_{64}\tau_{yz} + C_{65}\tau_{zx} + C_{66}\tau_{xy}
\end{aligned}$$

- An isotropic material has a stress-strain relationships that are independent of the orientation of the coordinate system at a point.
- A material is said to be homogenous if the material properties are the same at all points in the body. Alternatively, if the material constants C_{ij} are functions of the coordinates x, y, or z, then the material is called non-homogenous.

For Isotropic Materials: $$G = \frac{E}{2(1+\nu)}$$

Generalized Hooke's Law for Isotropic Materials

- The relationship between stresses and strains in three-dimensions is called the Generalized Hooke's Law.

$$\varepsilon_{xx} = [\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})]/E$$
$$\varepsilon_{yy} = [\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx})]/E$$
$$\varepsilon_{zz} = [\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})]/E$$

$$\gamma_{xy} = \tau_{xy}/G$$
$$\gamma_{yz} = \tau_{yz}/G$$
$$\gamma_{zx} = \tau_{zx}/G$$

$$G = \frac{E}{2(1+\nu)}$$

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \end{Bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix}\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \end{Bmatrix}$$

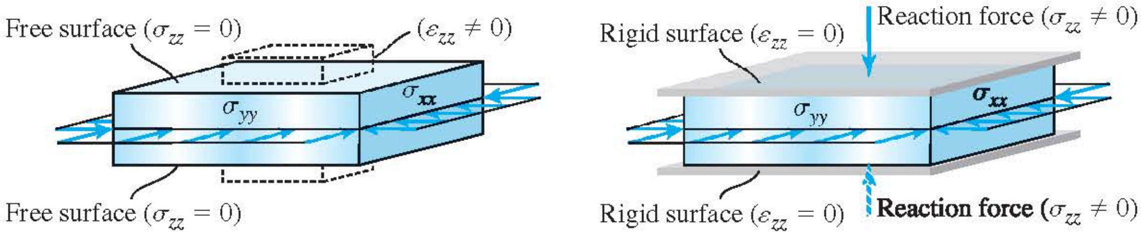
Plane Stress and Plane Strain

$$\text{Plane Stress} \longrightarrow \begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{Generalized Hooke's Law}} \begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & 0 \\ \gamma_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} = -\frac{\nu}{E}(\sigma_{xx} + \sigma_{yy}) \end{bmatrix}$$

$$\text{Plane Strain} \longrightarrow \begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & 0 \\ \gamma_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{Generalized Hooke's Law}} \begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) \end{bmatrix}$$

Plane Stress

Plane Strain



(a) (b)

C3.5 A 2in x 2 in square with a circle inscribed is stressed as shown Fig. C3.5. The plate material has a Modulus of Elasticity of E = 10,000 ksi and a Poisson's ratio ν = 0.25. Determine the major and minor axis of the ellipse formed due to deformation assuming (a) plane stress. (b) plane strain.

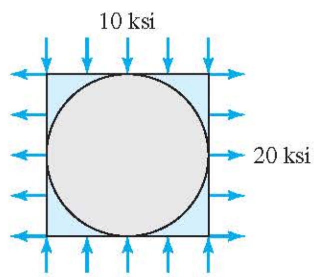


Fig. C3.5

Class Problem 3.1

The stress components at a point are as given.
Determine ε_{xx} assuming (a) Plane stress (b) Plane strain

$$\sigma_{xx} = 100 \ MPa(T)$$
$$\sigma_{yy} = 200 \ MPa(C)$$
$$\tau_{xy} = -125 \ MPa$$

$$E = 200 \ GPa$$
$$\nu = 0.25$$

Common Limitations to Theories in Chapters 4-7

- The length of the member is significantly greater (approximately 10 times) then the greatest dimension in the cross-section.
- We are away from regions of stress concentration, where displacements and stresses can be three-dimensional.
- The variation of external loads or changes in the cross-sectional area is gradual except in regions of stress concentration.
- The external loads are such that the axial, torsion and bending problems can be studied individually.