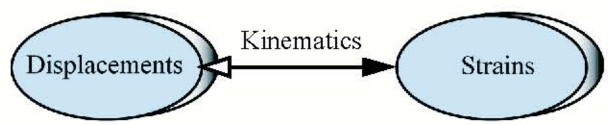
Strain

- Relating strains to displacements is a problem in geometry.



Learning objectives

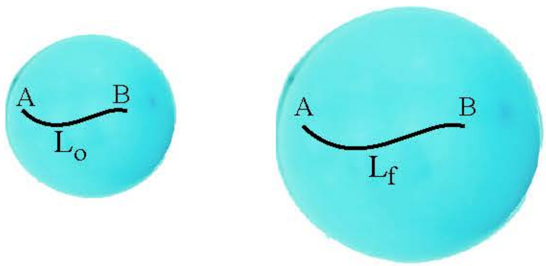
- Understand the concept of strain.

- Understand the use of approximate deformed shape for calculating strains from displacements.

Preliminary Definitions

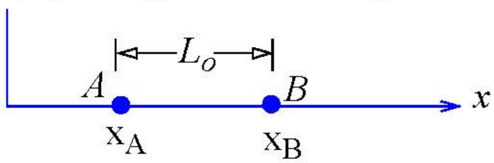
- The total movement of a point with respect to a fixed reference coordinates is called *displacement*.
- The relative movement of a point with respect to another point on the body is called *deformation*.
- *Lagrangian strain* is computed from deformation by using the original undeformed geometry as the reference geometry.
- *Eulerian strain* is computed from deformation by using the final deformed geometry as the reference geometry.

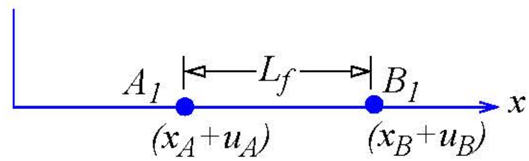
Average Normal Strain



$$\varepsilon_{av} = \frac{L_f - L_o}{L_o} = \frac{\delta}{L_o}$$

- Elongations ($L_f > L_o$) result in *positive* normal strains. Contractions ($L_f < L_o$) result in *negative* normal strains.





$$L_0 = x_B - x_A$$

$$L_f = (x_B + u_B) - (x_A + u_A) = L_o + (u_B -$$

$$\varepsilon_{av} = \frac{u_B - u_A}{x_B - x_A}$$

Units of average normal strain

- To differentiate average strain from strain at a point.
- in/in, or cm/cm, or m/m
- percentage. 0.5% is equal to a strain of 0.005
- prefix: $\mu = 10^{-6}$. 1000 μ in / in is equal to a strain 0.001 in / in

C2.1 Due to the application of the forces in Fig. C2.1, the displacement of the rigid plates in the x direction were observed as given below. Determine the axial strains in rods in sections AB, BC, and CD.

$$u_B = -1.8 \text{ mm} \qquad u_C = 0.7 \text{ mm} \qquad u_D = 3.7 \text{ mm}$$

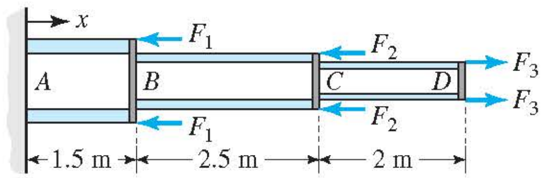


Fig. C2.1

Average shear strain

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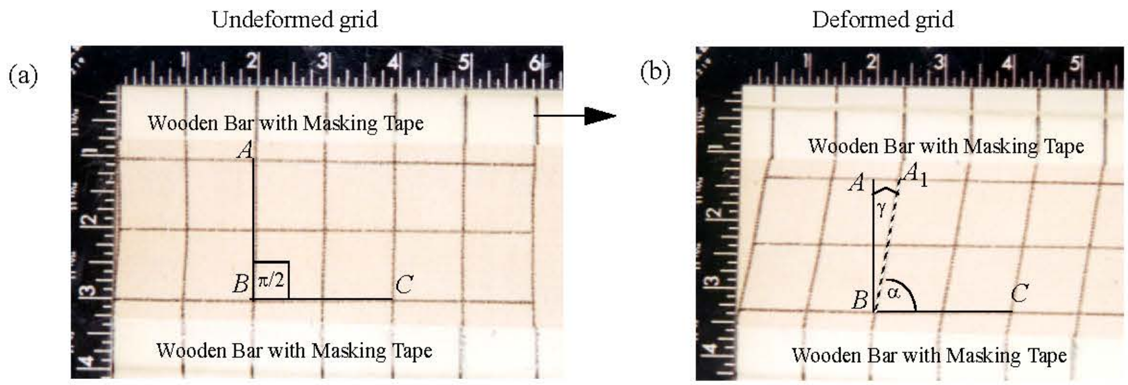


$$\gamma_{av} = \frac{\pi}{2} - \alpha$$

- Decreases in the angle $(\alpha < \pi / 2)$ result in *positive* shear strain. Increase in the angle $(\alpha > \pi / 2)$ result in *negative* shear strain

Units of average shear strain

- To differentiate average strain from strain at a point.
- rad
- prefix: $\mu = 10^{-6}$. 1000 μ rad is equal to a strain 0.001 rad

C2.2 A thin triangular plate ABC forms a right angle at point A. During deformation, point A moves vertically down by δ_A. Determine the average shear strain at point A.

$$\delta_A = 0.008 \quad in$$

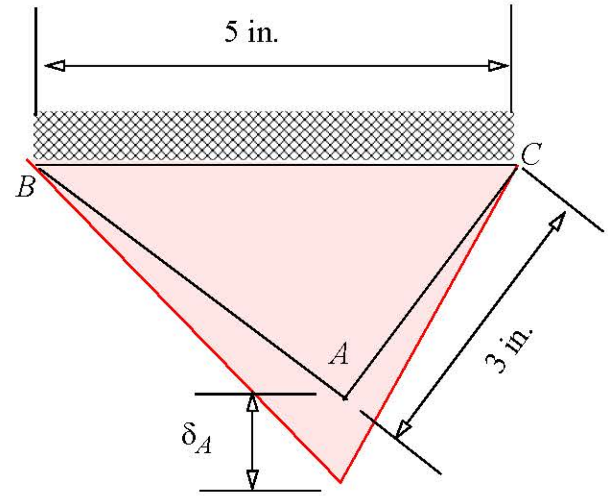
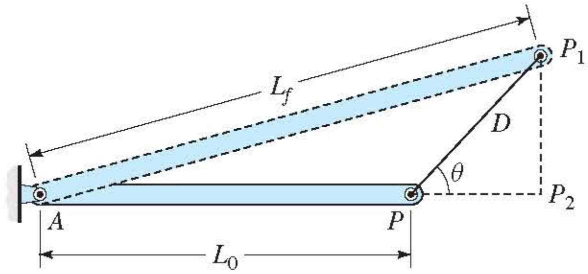


Fig. C2.2

Small Strain Approximation



$$L_f = \sqrt{L_o^2 + D^2 + 2L_oD\cos\theta}$$

$$L_f = L_o\sqrt{1 + \left(\frac{D}{L_o}\right)^2 + 2\left(\frac{D}{L_o}\right)\cos\theta}$$

$$\varepsilon = \frac{L_f - L_o}{L_o} = \sqrt{1 + \left(\frac{D}{L_o}\right)^2 + 2\left(\frac{D}{L_o}\right)\cos\theta} - 1 \qquad \textbf{2.5}$$

$$\varepsilon_{small} = \frac{D\cos\theta}{L_o} \qquad \textbf{2.6}$$

ε_{small} Eq. 2.6	ε Eq. 2.5	% error
1.0	1.23607	19.1
0.5	0.58114	14.0
0.1	0.10454	4.3
0.05	0.005119	2.32
0.01	0.01005	0.49
0.005	0.00501	0.25

- Small-strain approximation may be used for strains less than 0.01
- Small normal strains are calculated by using the deformation component in the original direction of the line element regardless of the orientation of the deformed line element.
- In small shear strain (γ) calculations the following approximation may be used for the trigonometric functions: $\tan\gamma \approx \gamma$ $\sin\gamma \approx \gamma$ $\cos\gamma \approx 1$
- Small-strain calculations result in linear deformation analysis.
- Drawing approximate deformed shape is very important in analysis of small strains.

C2.3 A roller at P slides in a slot as shown. Determine the deformation in bar AP and bar BP by using small strain approximation.

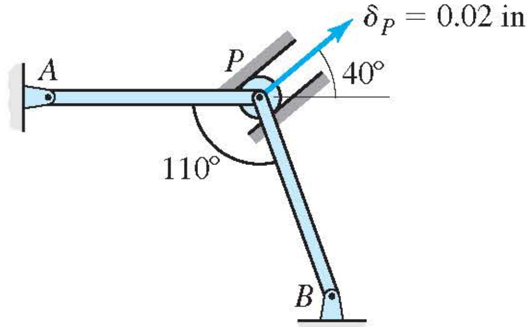
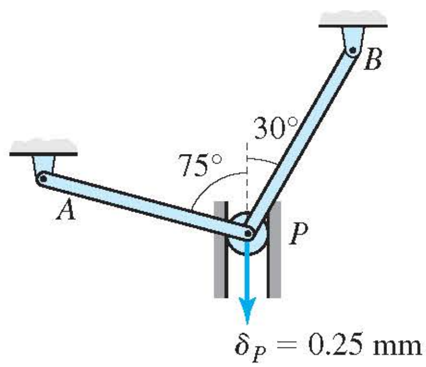


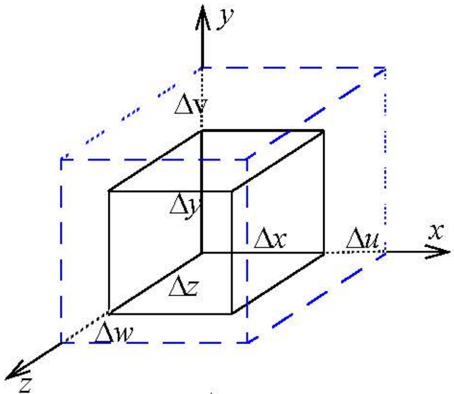
Fig. C2.3

Class Problem 2.1

Draw an approximate exaggerated deformed shape.
Using small strain approximation write equations relating δ_{AP} and δ_{BP} to δ_P.



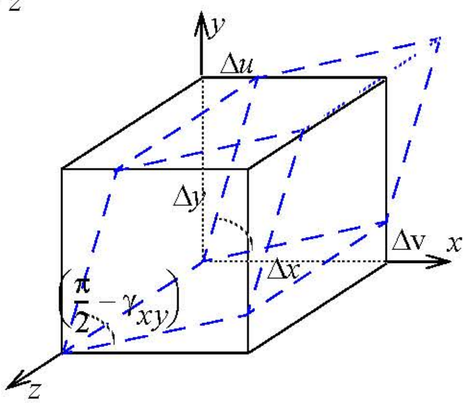
Strain Components



$$\varepsilon_{xx} = \frac{\Delta u}{\Delta x}$$

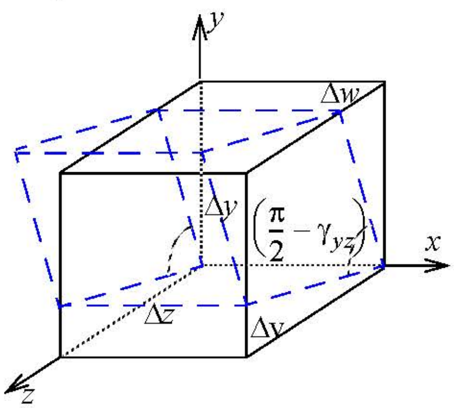
$$\varepsilon_{yy} = \frac{\Delta v}{\Delta y}$$

$$\varepsilon_{zz} = \frac{\Delta w}{\Delta z}$$



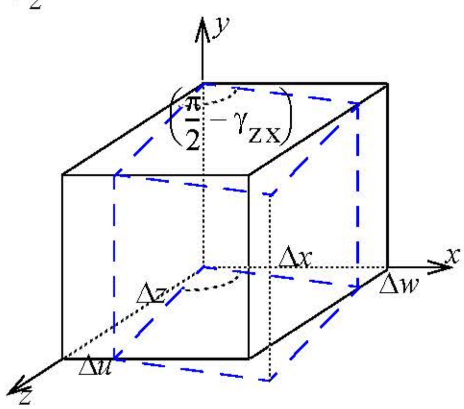
$$\gamma_{xy} = \frac{\Delta u}{\Delta y} + \frac{\Delta v}{\Delta x}$$

$$\gamma_{yx} = \frac{\Delta v}{\Delta x} + \frac{\Delta u}{\Delta y} = \gamma_{xy}$$



$$\gamma_{yz} = \frac{\Delta v}{\Delta z} + \frac{\Delta w}{\Delta y}$$

$$\gamma_{zy} = \frac{\Delta w}{\Delta y} + \frac{\Delta v}{\Delta z} = \gamma_{yz}$$



$$\gamma_{zx} = \frac{\Delta w}{\Delta x} + \frac{\Delta u}{\Delta z}$$

$$\gamma_{xz} = \frac{\Delta u}{\Delta z} + \frac{\Delta w}{\Delta x} = \gamma_{zx}$$

Engineering Strain

Engineering strain matrix

$$\begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & \gamma_{xz} \\ \gamma_{yx} & \varepsilon_{yy} & \gamma_{yz} \\ \gamma_{zx} & \gamma_{zy} & \varepsilon_{zz} \end{bmatrix}$$

Plane strain matrix

$$\begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & 0 \\ \gamma_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Strain at a point

$$\varepsilon_{xx} = \lim_{\Delta x \to 0}\left(\frac{\Delta u}{\Delta x}\right) = \frac{\partial u}{\partial x}$$

$$\gamma_{xy} = \gamma_{yx} = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}}\left(\frac{\Delta u}{\Delta y} + \frac{\Delta \mathrm{v}}{\Delta x}\right) = \frac{\partial u}{\partial y} + \frac{\partial \mathrm{v}}{\partial x}$$

$$\varepsilon_{yy} = \frac{\partial \mathrm{v}}{\partial y} \qquad \gamma_{yz} = \gamma_{zy} = \frac{\partial \mathrm{v}}{\partial z} + \frac{\partial w}{\partial y}$$

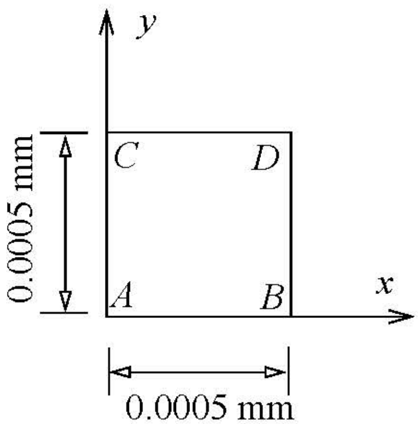
$$\varepsilon_{zz} = \frac{\partial w}{\partial z} \qquad \gamma_{zx} = \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

- tensor normal strains = engineering normal strains
- tensor shear strains = (engineering shear strains)/ 2

Strain at a Point on a Line

$$\varepsilon_{xx} = \frac{du(x)}{dx}$$

C2.4 Displacements u and v in the x and y directions respectively were measured by Moire Interferometry method at many points on a body. Displacements of four points on a body are given below. Determine the average values of strain components $\varepsilon_{xx},\ \varepsilon_{yy},\ and\ \gamma_{xy}$ at point A shown in Fig. C2.4.



$$u_A = 0 \qquad v_A = 0$$
$$u_B = 0.625\,\mu mm \qquad v_B = -0.3125\,\mu mm$$
$$u_C = -0.500\,\mu mm \qquad v_C = -0.5625\,\mu mm$$
$$u_D = 0.250\,\mu mm \qquad v_D = -1.125\,\mu mm$$

Fig. C2.4

C2.5 The axial displacement in a quadratic one-dimensional finite element is as given below. The displacements of nodes 1, 2, and 3 are u_1, u_2, and u_3, respectively

$$u(x) = \frac{u_1}{2a^2}(x-a)(x-2a) - \frac{u_2}{a^2}(x)(x-2a) + \frac{u_3}{2a^2}(x)(x-a)$$

Determine the strain at Node 2.