FORMULA SHEET

$\sigma_{ij} = \lim_{\Delta A_i \to 0}\left(\frac{\Delta F_j}{\Delta A_i}\right)$ $\sigma_{nn} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$ $\tau_{nt} = -\sigma_{xx}\cos\theta\sin\theta + \sigma_{yy}\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$ $\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_{xx} - \sigma_{yy})}$

$\sigma_{1,2} = \frac{(\sigma_{xx} + \sigma_{yy})}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$ $\varepsilon_{nn} = \varepsilon_{xx}\cos^2\theta + \varepsilon_{yy}\sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$ $\gamma_{nt} = -2\varepsilon_{xx}\sin\theta\cos\theta + 2\varepsilon_{yy}\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$

$\sigma_{nn} = \{n\}^T[\sigma]\{n\}$ $\tau_{nt} = \{t\}^T[\sigma]\{n\}$ $\sigma_{tt} = \{t\}^T[\sigma]\{t\}$ $\{S\} = [\sigma]\{n\}$

$\sigma_p^3 - I_1\sigma_p^2 + I_2\sigma_p - I_3 = 0$ $I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$ $I_2 = \begin{vmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{vmatrix} + \begin{vmatrix} \sigma_{yy} & \tau_{yz} \\ \tau_{zy} & \sigma_{zz} \end{vmatrix} + \begin{vmatrix} \sigma_{xx} & \tau_{xz} \\ \tau_{zx} & \sigma_{zz} \end{vmatrix}$ $I_3 = \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix}$

$x^3 - I_1x^2 + I_2x - I_3 = 0$ $x_1 = 2A\cos\alpha + I_1/3$ $x_{2,3} = -2A\cos(\alpha \pm 60^o) + I_1/3$ $A = \sqrt{(I_1/3)^2 - I_2/3}$ $\cos 3\alpha = [2(I_1/3)^3 - (I_1/3)I_2 + I_3]/(2A^3)$

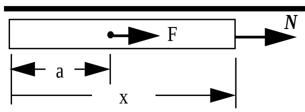
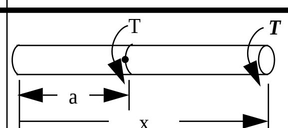
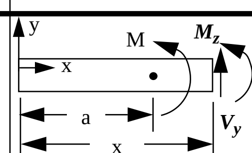
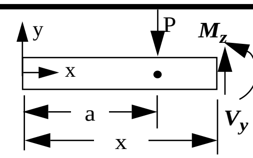
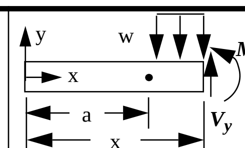
$\sigma_{oct} = (\sigma_1 + \sigma_2 + \sigma_3)/3$ $\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$ $\varepsilon_{xx} = \frac{\partial u}{\partial x}$ $\varepsilon_{yy} = \frac{\partial v}{\partial y}$ $\varepsilon_{zz} = \frac{\partial w}{\partial z}$ $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ $\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$ $\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$

$\varepsilon_{xx} = [\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})]/E + \alpha\Delta T$ $\gamma_{xy} = \tau_{xy}/G$ $G = E/[2(1+\nu)]$ $\sigma_{xx} = [\varepsilon_{xx} + \nu\varepsilon_{yy}]\frac{E}{(1-\nu^2)}$ $\varepsilon_{zz} = -\left(\frac{\nu}{1-\nu}\right)(\varepsilon_{xx} + \varepsilon_{yy})$ $\sigma_{xx} = \frac{E[(1-\nu)\varepsilon_{xx} + \nu\varepsilon_{yy}]}{(1-2\nu)(1+\nu)}$

$\varepsilon_{xx} = \frac{\sigma_{xx}}{E_x} - \frac{\nu_{yx}}{E_y}\sigma_{yy}$ $\gamma_{xy} = \frac{\tau_{xy}}{G_{xy}}$ $\frac{\nu_{yx}}{E_y} = \frac{\nu_{xy}}{E_x}$ $\sigma_{von} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$ $\left|\frac{\sigma_2}{\sigma_C} - \frac{\sigma_1}{\sigma_T}\right| \le 1$ $\tau_{max} = \left|max\left(\frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_2 - \sigma_3}{2}, \frac{\sigma_3 - \sigma_1}{2}\right)\right|$

$K_I = \sigma_{nom}\sqrt{\pi a}$ $K_{II} = \tau_{nom}\sqrt{\pi a}$ $K_{equiv} = \sqrt{K_I^2 + K_{II}^2}$

$\sigma = \begin{cases} \sigma_{yield} & \varepsilon \ge \varepsilon_{yield} \\ E\varepsilon & -\varepsilon_{yield} \le \varepsilon \le \varepsilon_{yield} \\ -\sigma_{yield} & \varepsilon \le -\varepsilon_{yield} \end{cases}$ $\sigma = \begin{cases} \sigma_{yield} + E_2(\varepsilon - \varepsilon_{yield}) & \varepsilon \ge \varepsilon_{yield} \\ E_1\varepsilon & -\varepsilon_{yield} \le \varepsilon \le \varepsilon_{yield} \\ -\sigma_{yield} + E_2(\varepsilon + \varepsilon_{yield}) & \varepsilon \ge \varepsilon_{yield} \end{cases}$ $\sigma = \begin{cases} E\varepsilon^n & \varepsilon \ge 0 \\ -E(-\varepsilon)^n & \varepsilon < 0 \end{cases}$

Axial (Rods)	Torsion (Shafts)	Bending (Beams)		
				
$\boldsymbol{N} = -F\langle x-a\rangle^0$ $p_x = F\langle x-a\rangle^{-1}$	$\boldsymbol{T} = -T\langle x-a\rangle^0$ $t = T\langle x-a\rangle^{-1}$	$\boldsymbol{M_z} = -M\langle x-a\rangle^0$ $p_y = -M\langle x-a\rangle^{-2}$	$\boldsymbol{M_z} = -P\langle x-a\rangle^1$ $p_y = -P\langle x-a\rangle^{-1}$	$\boldsymbol{M_z} = -w\frac{\langle x-a\rangle^2}{2}$ $p_y = -w\langle x-a\rangle^0$

$U_o = \frac{1}{2}\sigma\varepsilon$ $U_o = \frac{1}{2}[\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{zz}\varepsilon_{zz} + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx}]$

$(F=1)v_1(x_P) = \int_0^L \frac{\boldsymbol{M}_2(x)\boldsymbol{M}_1(x)}{EI}dx$ $(M=1)\frac{dv_1}{dx}(x_P) = \int_0^L \frac{\boldsymbol{M}_2(x)\boldsymbol{M}_1(x)}{EI}dx$ $v_1(x_P) = \frac{\partial \bar{U}_B}{\partial F}$ $\frac{dv_1}{dx}(x_P) = \frac{\partial \bar{U}_B}{\partial M}$

	Axial (Rods)	Torsion (Shafts)	Symmetric Bending (Beams)	Unsymmetric Bending
Displace-ments	$u(x, y, z) = u(x)$	$\phi(x, y, z) = \phi(x)$	$u(x, y, z) = -y\frac{dv}{dx} \quad v = v(x) \quad w = 0$	$u(x, y, z) = -y\frac{dv}{dx} - z\frac{dw}{dx} \quad v = v(x) \quad w = w(x)$
Strains	$\varepsilon_{xx} = \frac{du}{dx}$	$\gamma_{x\theta} = \rho\frac{d\phi}{dx}$	$\varepsilon_{xx} = -y\frac{d^2v}{dx^2}$	$\varepsilon_{xx} = -y\frac{d^2v}{dx^2} - z\frac{d^2w}{dx^2}$
Stresses	$\sigma_{xx} = E\varepsilon_{xx} = E\frac{du}{dx}$	$\tau_{x\theta} = G\gamma_{x\theta} = G\rho\frac{d\phi}{dx}$	$\sigma_{xx} = E\varepsilon_{xx} = -Ey\frac{d^2v}{dx^2} \quad \tau_{xy} \neq 0 \ll \sigma_{xx}$	$\sigma_{xx} = -Ey\frac{d^2v}{dx^2} - Ez\frac{d^2w}{dx^2} \quad \tau_{xy} \neq 0 \ll \sigma_{xx} \quad \tau_{xz} \neq 0 \ll \sigma_{xx}$
Internal Forces & Moments	$\boldsymbol{N} = \int_A \sigma_{xx}dA$	$\boldsymbol{T} = \int_A \rho\tau_{x\theta}dA$	$\boldsymbol{N} = \int_A \sigma_{xx}dA = 0$ $M_z = -\int_A y\sigma_{xx}dA \quad V_y = \int_A \tau_{xy}dA$	$\boldsymbol{N} = \int_A \sigma_{xx}dA = 0 \quad \boldsymbol{M}_z = -\int_A y\sigma_{xx}dA \quad \boldsymbol{M}_y = -\int_A z\sigma_{xx}dA$ $\boldsymbol{V}_y = \int_A \tau_{xy}dA \quad \boldsymbol{V}_z = \int_A \tau_{xz}dA$
	$\sigma_{xx} = \frac{\boldsymbol{N}}{A}$	$\tau_{x\theta} = \frac{\boldsymbol{T}\rho}{J}$	$\sigma_{xx} = -\left(\frac{\boldsymbol{M}_z y}{I_{zz}}\right)$ $q = \tau_{xs}t = -\left(\frac{\boldsymbol{V}_y Q_z}{I_{zz}}\right)$	$\sigma_{xx} = -\left(\frac{I_{yy}\boldsymbol{M}_z - I_{yz}\boldsymbol{M}_y}{I_{yy}I_{zz} - I_{yz}^2}\right)y - \left(\frac{I_{zz}\boldsymbol{M}_y - I_{yz}\boldsymbol{M}_z}{I_{yy}I_{zz} - I_{yz}^2}\right)z$ $q = \tau_{xs}t = -\left(\frac{I_{yy}Q_z - I_{yz}Q_y}{I_{yy}I_{zz} - I_{yz}^2}\right)\boldsymbol{V}_y - \left(\frac{I_{zz}Q_y - I_{yz}Q_z}{I_{yy}I_{zz} - I_{yz}^2}\right)\boldsymbol{V}_z$
	$\frac{du}{dx} = \frac{\boldsymbol{N}}{EA} \quad u_2 - u_1 = \frac{\boldsymbol{N}(x_2 - x_1)}{EA}$	$\frac{d\phi}{dx} = \frac{\boldsymbol{T}}{GJ} \quad \phi_2 - \phi_1 = \frac{\boldsymbol{T}(x_2 - x_1)}{GJ}$	$\frac{d^2v}{dx^2} = \frac{\boldsymbol{M}_z}{EI_{zz}} \quad v = \int\left[\int\frac{\boldsymbol{M}_z}{EI}dx\right]dx + C_1x + C_2$	$\frac{d^2v}{dx^2} = \frac{1}{E}\left(\frac{I_{yy}\boldsymbol{M}_z - I_{yz}\boldsymbol{M}_y}{I_{yy}I_{zz} - I_{yz}^2}\right) \quad \frac{d^2w}{dx^2} = \frac{1}{E}\left(\frac{I_{zz}\boldsymbol{M}_y - I_{yz}\boldsymbol{M}_z}{I_{yy}I_{zz} - I_{yz}^2}\right)$
	$(\sigma_{xx})_i = \frac{\boldsymbol{N}E_i}{\sum_{j=1}^{n} E_jA_j}$	$(\tau_{x\theta})_i = \frac{G_i\rho\boldsymbol{T}}{\left[\sum_{j=1}^{n} G_jJ_j\right]}$	$(\sigma_{xx})_i = -\left[\frac{E_i y\boldsymbol{M}_z}{\sum_{j=1}^{n} E_j(I_{zz})_j}\right] \quad q = \tau_{xs}t = -\left\{\frac{Q_{comp}\boldsymbol{V}_y}{\left[\sum_{j=1}^{n} E_j(I_{zz})_j\right]}\right\}$	
	$u_2 - u_1 = \frac{\boldsymbol{N}(x_2 - x_1)}{\sum E_jA_j}$	$\phi_2 - \phi_1 = \frac{\boldsymbol{T}(x_2 - x_1)}{[\sum G_jJ_j]}$	$v = \int\left[\int\frac{\boldsymbol{M}_z}{\sum E_j(I_{zz})_j}dx\right]dx + C_1x + C_2$	
	$\frac{d\boldsymbol{N}}{dx} = -p_x(x)$	$\frac{d\boldsymbol{T}}{dx} = -t(x)$	$\frac{d\boldsymbol{V}_y}{dx} = -p_y(x) \quad \frac{d\boldsymbol{M}_z}{dx} = -\boldsymbol{V}_y$	$\frac{d\boldsymbol{V}_y}{dx} = -p_y(x) \quad \frac{d\boldsymbol{M}_z}{dx} = -\boldsymbol{V}_y \quad \frac{d\boldsymbol{V}_z}{dx} = -p_z(x) \quad \frac{d\boldsymbol{M}_y}{dx} = -\boldsymbol{V}_z$
	$\frac{d}{dx}\left(EA\frac{du_o}{dx}\right) = -p_x(x)$	$\frac{d}{dx}\left(GJ\frac{d\phi}{dx}\right) = -t(x)$	$\frac{d^2}{dx^2}\left(EI_{zz}\frac{d^2v}{dx^2}\right) = p_y(x)$	
Strain Energy	$U_a = EA\left(\frac{du}{dx}\right)^2/2$	$U_t = GJ\left(\frac{d\phi}{dx}\right)^2/2$	$U_b = EI_{zz}\left(\frac{d^2v}{dx^2}\right)^2/2$	
C. Strain Energy	$\bar{U}_a = \boldsymbol{N}^2/(2EA)$	$\bar{U}_t = \boldsymbol{T}^2/(2GJ)$	$\bar{U}_b = \boldsymbol{M}_z^2/(2EI_{zz})$	