FORMULA SHEET

$$\sigma_{ij} = \lim_{\Delta A_i \to 0}\left(\frac{\Delta F_j}{\Delta A_i}\right) \qquad \sigma_{nn} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta \qquad \tau_{nt} = -\sigma_{xx}\cos\theta\sin\theta + \sigma_{yy}\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_{xx}-\sigma_{yy})} \qquad \sigma_{1,2} = \frac{(\sigma_{xx}+\sigma_{yy})}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} \qquad \varepsilon_{nn} = \varepsilon_{xx}\cos^2\theta + \varepsilon_{yy}\sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\gamma_{nt} = -2\varepsilon_{xx}\sin\theta\cos\theta + 2\varepsilon_{yy}\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\sigma_{nn} = \{n\}^T[\sigma]\{n\} \qquad \tau_{nt} = \{t\}^T[\sigma]\{n\} \qquad \sigma_{tt} = \{t\}^T[\sigma]\{t\} \qquad \{S\} = [\sigma]\{n\} \qquad \sigma_p^3 - I_1\sigma_p^2 + I_2\sigma_p - I_3 = 0$$

$$I_1 = \sigma_{xx}+\sigma_{yy}+\sigma_{zz} \qquad I_2 = \begin{vmatrix}\sigma_{xx} & \tau_{xy}\\ \tau_{yx} & \sigma_{yy}\end{vmatrix} + \begin{vmatrix}\sigma_{yy} & \tau_{yz}\\ \tau_{zy} & \sigma_{zz}\end{vmatrix} + \begin{vmatrix}\sigma_{xx} & \tau_{xz}\\ \tau_{zx} & \sigma_{zz}\end{vmatrix} \qquad I_3 = \begin{vmatrix}\sigma_{xx} & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \sigma_{yy} & \tau_{yz}\\ \tau_{zx} & \tau_{zy} & \sigma_{zz}\end{vmatrix} \qquad x^3 - I_1x^2 + I_2x - I_3 = 0$$

$$x_1 = 2A\cos\alpha + I_1/3 \qquad x_{2,3} = -2A\cos(\alpha \pm 60^o) + I_1/3 \qquad A = \sqrt{(I_1/3)^2 - I_2/3} \qquad \cos 3\alpha = [2(I_1/3)^3 - (I_1/3)I_2 + I_3]/(2A^3)$$

$$\sigma_{oct} = (\sigma_1+\sigma_2+\sigma_3)/3 \qquad \tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} \qquad \varepsilon_{xx} = \frac{\partial u}{\partial x} \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y} \qquad \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \qquad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \qquad \varepsilon_{rr} = \frac{\partial u_r}{\partial r} \qquad \varepsilon_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} \qquad \gamma_{r\theta} = \frac{1}{r}\frac{\partial u_r}{\partial\theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}$$

$$\varepsilon_{xx} = [\sigma_{xx} - \nu(\sigma_{yy}+\sigma_{zz})]/E + \alpha\Delta T \qquad \gamma_{xy} = \tau_{xy}/G \qquad G = E/[2(1+\nu)] \qquad \sigma_{xx} = [\varepsilon_{xx} + \nu\varepsilon_{yy}]\frac{E}{(1-\nu^2)} \qquad \varepsilon_{zz} = -\left(\frac{\nu}{1-\nu}\right)(\varepsilon_{xx}+\varepsilon_{yy})$$

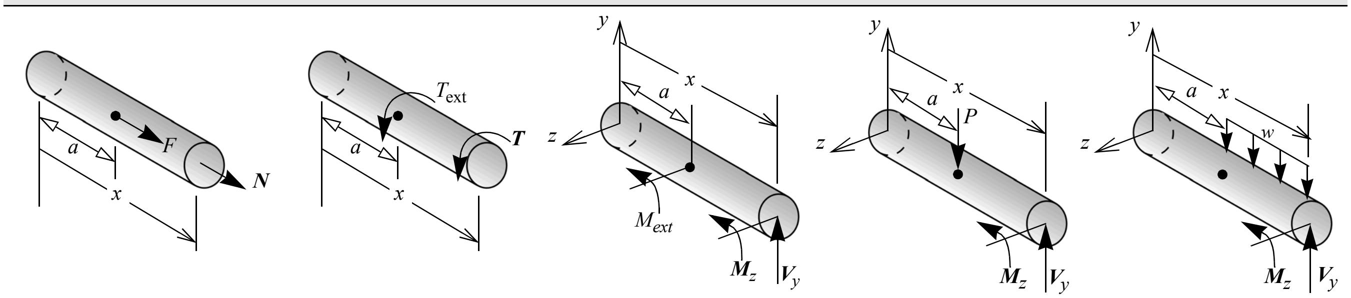
Plane strain $\begin{matrix}\nu \to \nu/(1+\nu)\\ G \to G\end{matrix}$ Plane stress $\qquad \sigma_{xx} = \dfrac{E[(1-\nu)\varepsilon_{xx} + \nu\varepsilon_{yy}]}{(1-2\nu)(1+\nu)} \qquad \varepsilon_{xx} = \dfrac{\sigma_{xx}}{E_x} - \dfrac{\nu_{yx}}{E_y}\sigma_{yy} \qquad \gamma_{xy} = \dfrac{\tau_{xy}}{G_{xy}} \qquad \dfrac{\nu_{yx}}{E_y} = \dfrac{\nu_{xy}}{E_x}$

$$\sigma_{von} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} \qquad \left|\frac{\sigma_2}{\sigma_C} - \frac{\sigma_1}{\sigma_T}\right| \le 1 \qquad \tau_{max} = \left|max\left(\frac{\sigma_1-\sigma_2}{2}, \frac{\sigma_2-\sigma_3}{2}, \frac{\sigma_3-\sigma_1}{2}\right)\right|$$

$$K_I = \sigma_{nom}\sqrt{\pi a} \qquad K_{II} = \tau_{nom}\sqrt{\pi a} \qquad K_{equiv} = \sqrt{K_I^2 + K_{II}^2}$$

$$\sigma = \begin{cases}\sigma_{yield} & \varepsilon \ge \varepsilon_{yield}\\ E\varepsilon & -\varepsilon_{yield} \le \varepsilon \le \varepsilon_{yield}\\ -\sigma_{yield} & \varepsilon \le -\varepsilon_{yield}\end{cases} \qquad \sigma = \begin{cases}\sigma_{yield} + E_2(\varepsilon - \varepsilon_{yield}) & \varepsilon \ge \varepsilon_{yield}\\ E_1\varepsilon & -\varepsilon_{yield} \le \varepsilon \le \varepsilon_{yield}\\ -\sigma_{yield} + E_2(\varepsilon + \varepsilon_{yield}) & \varepsilon \ge \varepsilon_{yield}\end{cases} \qquad \sigma = \begin{cases}E\varepsilon^n & \varepsilon \ge 0\\ -E(-\varepsilon)^n & \varepsilon < 0\end{cases}$$

Axial (Rods)	Torsion (Shafts)	Bending (Beams)		



$N = -F\langle x-a\rangle^0$	$T = -T\langle x-a\rangle^0$	$M_z = -M_{ext}\langle x-a\rangle^0$	$M_z = -P\langle x-a\rangle^1$	$M_z = -w\langle x-a\rangle^2/2$
$p_x = F\langle x-a\rangle^{-1}$	$t = T\langle x-a\rangle^{-1}$	$p_y = -M_{ext}\langle x-a\rangle^{-2}$	$p_y = -P\langle x-a\rangle^{-1}$	$p_y = -w\langle x-a\rangle^0$

$$U_o = \frac{1}{2}\sigma\varepsilon \qquad U_o = \frac{1}{2}[\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{zz}\varepsilon_{zz} + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx}]$$

$$(F=1)v_1(x_P) = \int_0^L \frac{M_2(x)M_1(x)}{EI}dx \qquad (M=1)\frac{dv_1}{dx}(x_P) = \int_0^L \frac{M_2(x)M_1(x)}{EI}dx \qquad v_1(x_P) = \frac{\partial \bar{U}_B}{\partial F} \qquad \frac{dv_1}{dx}(x_P) = \frac{\partial \bar{U}_B}{\partial M}$$

$$\frac{\partial^2\varepsilon_{xx}}{\partial y^2} + \frac{\partial^2\varepsilon_{yy}}{\partial x^2} = \frac{\partial^2\gamma_{xy}}{\partial x\partial y} \qquad \frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + F_x = 0 \qquad \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_{yy}}{\partial y} + F_y = 0 \qquad \frac{\partial\sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} + \frac{\sigma_{rr}-\sigma_{\theta\theta}}{r} + F_r = 0 \qquad \frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta} + \frac{\partial\tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} + F_\theta = 0$$

$$\sigma_{xx} = \frac{\partial^2\psi}{\partial y^2} \qquad \sigma_{yy} = \frac{\partial^2\psi}{\partial x^2} \qquad \tau_{xy} = -\left(\frac{\partial^2\psi}{\partial x\partial y}\right) \qquad \nabla^4\psi = \frac{\partial^4\psi}{\partial x^4} + 2\frac{\partial^4\psi}{\partial x^2\partial y^2} + \frac{\partial^4\psi}{\partial y^4} = 0$$

$$\sigma_{rr} = \frac{1}{R_o^2 - R_i^2}\left[-(p_oR_o^2 - p_iR_i^2) - \frac{R_i^2R_o^2(p_i - p_o)}{r^2}\right] \qquad \sigma_{\theta\theta} = \frac{1}{R_o^2 - R_i^2}\left[-(p_oR_o^2 - p_iR_i^2) + \frac{R_i^2R_o^2(p_i - p_o)}{r^2}\right]$$

Axial	Torsion	Symmetric Bending (Straight)	Symmetric Bending (Curved)	Unsymmetric Bending (Straight)
$u = u_o(x)$	$\phi = \phi(x)$	$u = -y\frac{dv}{dx}$ $v = v(x)$		$u(x, y, z) = -y\frac{dv}{dx} - z\frac{dw}{dx}$ $v = v(x) \quad w = w(x)$
$\varepsilon_{xx} = \frac{du_o}{dx}(x)$	$\gamma_{x\theta} = \rho\frac{d\phi}{dx}(x)$	$\varepsilon_{xx} = -y\frac{d^2v}{dx^2}(x)$	$\varepsilon_{\theta\theta} = -\left(1 - \frac{R}{\rho}\right)\psi$	$\varepsilon_{xx} = -y\frac{d^2v}{dx^2} - z\frac{d^2w}{dx^2}$
$\sigma_{xx} = E\frac{du_o}{dx}(x)$	$\tau_{x\theta} = G\rho\frac{d\phi}{dx}(x)$	$\sigma_{xx} = -Ey\frac{d^2v}{dx^2}(x)$ $\lvert\tau_{max}\rvert \ll \lvert\sigma_{max}\rvert$	$\sigma_{\theta\theta} = -E\left(1 - \frac{R}{\rho}\right)\psi$	$\sigma_{xx} = -Ey\frac{d^2v}{dx^2} - Ez\frac{d^2w}{dx^2}$ $\lvert\tau_{max}\rvert \ll \lvert\sigma_{max}\rvert$
$\boldsymbol{N} = \int_A \sigma_{xx} dA$ $\boldsymbol{M}_z = 0$ $\boldsymbol{M}_y = 0$	$\boldsymbol{T} = \int_A \rho\tau_{x\theta} dA$	$\boldsymbol{N} = 0$ $\boldsymbol{M}_z = -\int_A y\sigma_{xx} dA$ $\boldsymbol{V}_y = \int_A \tau_{xy} dA$	$\boldsymbol{N} = 0$ $\boldsymbol{M}_z = -\int_A \rho\sigma_{\theta\theta} dA$	$\boldsymbol{N} = \int_A \sigma_{xx} dA = 0$; $\boldsymbol{M}_z = -\int_A y\sigma_{xx} dA$; $\boldsymbol{M}_y = -\int_A z\sigma_{xx} dA$; $\boldsymbol{V}_y = \int_A \tau_{xy} dA$; $\boldsymbol{V}_z = \int_A \tau_{xz} dA$
$\int_A y dA = 0$		$\int_A y dA = 0$	$R = A/\int_A \frac{dA}{\rho}$	$\int_A y dA = 0$; $\int_A z dA = 0$
$\frac{du_o}{dx} = \frac{\boldsymbol{N}}{EA}$	$\frac{d\phi}{dx} = \frac{\boldsymbol{T}}{GJ}$	$\frac{d^2v}{dx^2} = \frac{\boldsymbol{M}_z}{EI_{zz}}$	$\psi = \frac{\boldsymbol{M}_z}{EA(r_c - R)}$	$\frac{d^2v}{dx^2} = \frac{1}{E}\left(\frac{I_{yy}\boldsymbol{M}_z - I_{yz}\boldsymbol{M}_y}{I_{yy}I_{zz} - I_{yz}^2}\right)$ $\frac{d^2w}{dx^2} = \frac{1}{E}\left(\frac{I_{zz}\boldsymbol{M}_y - I_{yz}\boldsymbol{M}_z}{I_{yy}I_{zz} - I_{yz}^2}\right)$
$\sigma_{xx} = \frac{\boldsymbol{N}}{A}$	$\tau_{x\theta} = \frac{\boldsymbol{T}\rho}{J}$	$\sigma_{xx} = -\left(\frac{\boldsymbol{M}_z y}{I_{zz}}\right)$ $\tau_{xs} = -\left(\frac{\boldsymbol{V}_y Q_z}{I_{zz}t}\right)$	$\sigma_{\theta\theta} = -\left[\frac{\boldsymbol{M}_z\left(1 - \frac{R}{\rho}\right)}{A(r_c - R)}\right]$	$\sigma_{xx} = -\left[\frac{I_{yy}\boldsymbol{M}_z - I_{yz}\boldsymbol{M}_y}{I_{yy}I_{zz} - I_{yz}^2}\right]y - \left[\frac{I_{zz}\boldsymbol{M}_y - I_{yz}\boldsymbol{M}_z}{I_{yy}I_{zz} - I_{yz}^2}\right]z$ $\tau_{xs} = -\left[\frac{I_{yy}Q_z - I_{yz}Q_y}{\left(I_{yy}I_{zz} - I_{yz}^2\right)t}\right]\boldsymbol{V}_y - \left[\frac{I_{zz}Q_y - I_{yz}Q_z}{\left(I_{yy}I_{zz} - I_{yz}^2\right)t}\right]\boldsymbol{V}_z$
$\frac{d\boldsymbol{N}}{dx} = -p_x(x)$	$\frac{d\boldsymbol{T}}{dx} = -t(x)$	$\frac{d\boldsymbol{V}_y}{dx} = -p_y(x)$; $\frac{d\boldsymbol{M}_z}{dx} = -\boldsymbol{V}_y$		$\frac{d\boldsymbol{V}_y}{dx} = -p_y(x)$; $\frac{d\boldsymbol{M}_z}{dx} = -\boldsymbol{V}_y$; $\frac{d\boldsymbol{V}_z}{dx} = -p_z(x) \quad \frac{d\boldsymbol{M}_y}{dx} = -\boldsymbol{V}_z$
$\frac{d}{dx}\left(EA\frac{du_o}{dx}\right) = -p_x(x)$	$\frac{d}{dx}\left(GJ\frac{d\phi}{dx}\right) = -t(x)$	$\frac{d^2}{dx^2}\left(EI_{zz}\frac{d^2v}{dx^2}\right) = p_y(x)$		$\frac{d^4v}{dx^4} = \frac{1}{E}\left(\frac{I_{yy}p_y - I_{yz}p_z}{I_{yy}I_{zz} - I_{yz}^2}\right)$; $\frac{d^4w}{dx^4} = \frac{1}{E}\left(\frac{I_{zz}p_z - I_{yz}p_y}{I_{yy}I_{zz} - I_{yz}^2}\right)$
$\frac{du_o}{dx} = \frac{\boldsymbol{N}}{\sum E_jA_j}$ $(\sigma_{xx})_i = \frac{\boldsymbol{N}E_i}{\sum E_jA_j}$	$\frac{d\phi}{dx} = \frac{\boldsymbol{T}}{\sum G_jJ_j}$ $(\tau_{x\theta})_i = \frac{G_i\rho\boldsymbol{T}}{\sum G_jJ_j}$	$\frac{d^2v}{dx^2} = \frac{\boldsymbol{M}_z}{\sum E_j(I_{zz})_j}$ $(\sigma_{xx})_i = -\left[\frac{E_iy\boldsymbol{M}_z}{\sum E_j(I_{zz})_j}\right]$ $\tau_{xs} = -\left[\frac{Q_{comp}\boldsymbol{V}_y}{t\sum E_j(I_{zz})_j}\right]$		
$U_a = \frac{1}{2}EA\left(\frac{du}{dx}\right)^2$	$U_t = \frac{1}{2}GJ\left(\frac{d\phi}{dx}\right)^2$	$U_b = \frac{1}{2}EI_{zz}\left(\frac{d^2v}{dx^2}\right)^2$		
$\bar{U}_a = \frac{\boldsymbol{N}^2}{2EA}$	$\bar{U}_t = \frac{\boldsymbol{T}^2}{2GJ}$	$\bar{U}_b = \frac{\boldsymbol{M}_z^2}{2EI_{zz}}$		