Formula Sheet Fall

$$\int_0^{\pi} sin(i\theta)\,sin(j\theta)\,d\theta = \begin{cases} 0 & i \neq j \\ \pi/2 & i = j \end{cases} \qquad \int_0^{\pi} cos(i\theta)\,cos(j\theta)\,d\theta = \begin{cases} 0 & i \neq j \\ \pi/2 & i = j \end{cases}$$

$$\iint_A f\frac{\partial g}{\partial x}dxdy = \oint_\Gamma fg\,dy - \iint_A g\frac{\partial f}{\partial x}dxdy \qquad \iint_A f\frac{\partial g}{\partial y}dxdy = -\oint_\Gamma fg\,dx - \iint_A g\frac{\partial f}{\partial y}dxdy \qquad \iiint_T \frac{\partial f}{\partial x_i}g\,dV = \iint_S fgn_i\,dA - \iiint_T f\frac{\partial g}{\partial x_i}dV$$

$$[K^{(e)}] = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \{R^{(e)}\} = \frac{p_oL}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + \begin{Bmatrix} F_1^{(e)} \\ F_2^{(e)} \end{Bmatrix} \qquad [K^{(e)}] = \frac{EA}{3L}\begin{bmatrix} 7 & -8 & 1 \\ -8 & 16 & -8 \\ 1 & -8 & 7 \end{bmatrix} \qquad \{R\} = \frac{p_oL}{6}\begin{Bmatrix} 1 \\ 4 \\ 1 \end{Bmatrix} + \begin{Bmatrix} F_1^{(e)} \\ 0 \\ F_3^{(e)} \end{Bmatrix}$$

$$[K_G^{(e)}] = [T]^T[b^{(e)}][T] \qquad [R_G^{(e)}] = [T]^T[R^{(e)}]\,;\; [T] = \begin{bmatrix} cos\theta & sin\theta & 0 & 0 \\ 0 & 0 & cos\theta & sin\theta \end{bmatrix};\; [K^{(e)}] = \frac{2EI}{L^3}\begin{bmatrix} 6 & 3L & -6 & 3L \\ 3L & 2L^2 & -3L & L^2 \\ -6 & -3L & 6 & -3L \\ 3L & L^2 & -3L & 2L^2 \end{bmatrix};$$

$$f_1(x) = 1 - 3\left(\frac{x-x_1}{L}\right)^2 + 2\left(\frac{x-x_1}{L}\right)^3 \qquad f_2(x) = L\left[\left(\frac{x-x_1}{L}\right) - 2\left(\frac{x-x_1}{L}\right)^2 + \left(\frac{x-x_1}{L}\right)^3\right]$$

$$f_3(x) = 3\left(\frac{x-x_1}{L}\right)^2 - 2\left(\frac{x-x_1}{L}\right)^3 \qquad f_4(x) = L\left[-\left(\frac{x-x_1}{L}\right)^2 + \left(\frac{x-x_1}{L}\right)^3\right]\,;\; \{R^{(e)}\} = \frac{p_0L}{12}\begin{Bmatrix} 6 \\ L \\ 6 \\ -L \end{Bmatrix} + \begin{Bmatrix} F_1^{(e)} \\ M_1^{(e)} \\ F_2^{(e)} \\ M_2^{(e)} \end{Bmatrix}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -f \qquad B(v, u) = \iint_\Omega \left[\frac{\partial v\partial u}{\partial x\partial x} + \frac{\partial v\partial u}{\partial y\partial y}\right]dxdy \qquad l(v) = \iint_\Omega vf\,dxdy + \oint_\Gamma vq_n\,ds \qquad q_n = \frac{\partial u}{\partial n} = \frac{\partial u}{\partial x}n_x + \frac{\partial u}{\partial y}n_y$$

$$L_I = \frac{1}{2A}[(x-x_K)(y_J-y_K) - (y-y_K)(x_J-x_K)]\,;\; L_I = \frac{1}{6V}[(x-x_J)(y_{KJ}z_{MJ} - z_{KJ}y_{MJ}) + (y-y_J)(z_{KJ}x_{MJ} - x_{MJ}z_{KJ}) + (z-z_J)(x_{KJ}y_{MJ} - y_{KJ}x_{MJ})]$$

$$\int_a^b L_I^m L_J^n L_K^p\,ds = \frac{(b-a)\,m!n!p!}{(m+n+p+1)!} \qquad \iint_A L_I^m L_J^n L_K^p\,dxdy = \frac{(2A)\,m!n!p!}{(m+n+p+2)!} \qquad \iiint_V L_I^m L_J^n L_K^p L_M^q\,dxdydz = \frac{(6V)\,m!n!p!q!}{(m+n+p+q+3)!}$$

$$K_{ij}^{(e)} = \int_{-1}^{1}\int_{-1}^{1}\begin{Bmatrix} \frac{\partial\phi_i}{\partial\xi} \\ \frac{\partial\phi_i}{\partial\eta} \end{Bmatrix}^T [[J]^{-1}]^T[J]^{-1}\begin{Bmatrix} \frac{\partial\phi_j}{\partial\xi} \\ \frac{\partial\phi_j}{\partial\eta} \end{Bmatrix}|J|\,d\xi d\eta \qquad R_i^{(e)} = \int_{-1}^{1}\int_{-1}^{1}\phi_i f|J|\,d\xi d\eta + \oint_\Gamma \phi_i q_n\,ds \qquad [J] = \begin{bmatrix} \sum_i x_i^{(e)}\frac{\partial\psi_i}{\partial\xi} & \sum_i y_i^{(e)}\frac{\partial\psi_i}{\partial\xi} \\ \sum_i x_i^{(e)}\frac{\partial\psi_i}{\partial\eta} & \sum_i y_i^{(e)}\frac{\partial\psi_i}{\partial\eta} \end{bmatrix}$$

Base points ξ_i	n	Weights w_i
0.0	One point formula	w_1= 2.0
$\pm(1/\sqrt{3}) = \pm 0.5774$	Two point formula	w_1= w_2= 1.0
0.0; $\pm\sqrt{0.6} = \pm 0.7746$	Three point formula	w_1= (8/9)=0.8889; w_2= w_3= (5/9)=0.5556

$$[M]\begin{Bmatrix} \frac{d^2u}{dt^2} \end{Bmatrix} + [C]\begin{Bmatrix} \frac{du}{dt} \end{Bmatrix} + [K]\{u\} = \{R\}$$