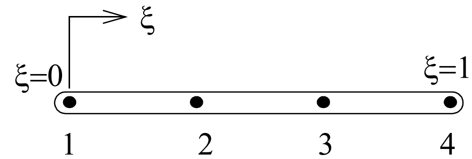
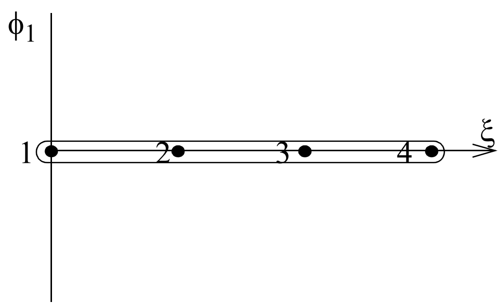
EXAM 2

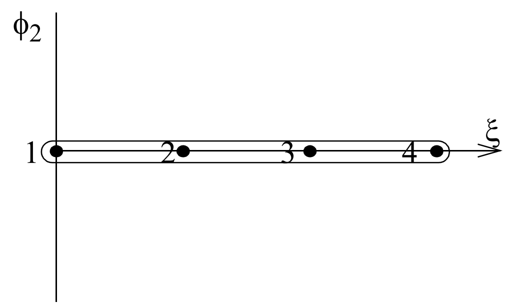
1.

(i) Using the natural coordinate (ξ) shown, write the Lagrange interpolation function associated with nodes 3 and 4 for the cubic element. The nodes are evenly spaced.

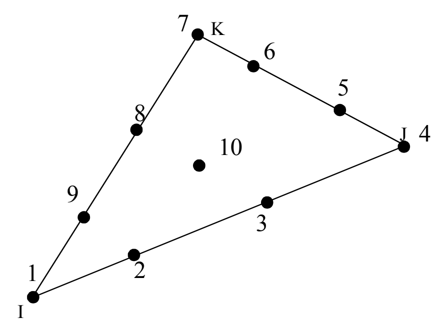


(ii) Plot the approximate shape for the Lagrange interpolation functions associated with nodes 1 and 2.





(iii) Using Area coordinates L_I, L_J, L_K write the Lagrange interpolation function associated with nodes 5 and 7. The nodes are uniformly spaced.



(iv) Write the expression for $\frac{\partial \phi_7}{\partial x}$ from the Lagrange interpolation function you obtained in part (iii).

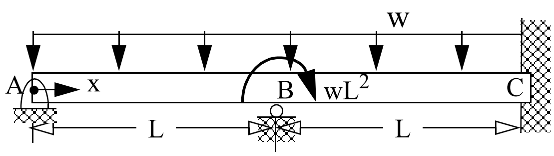
(v) Using a 3 point Gauss quadrature evaluate the following integral : $I = \int_{-1}^{1} \frac{(1+\xi^2)}{(1+\xi^4)} d\xi$

CIRCLE THE CORRECT ANSWERS BELOW

(vi) In FEM we are using, the secondary variables and related quantities are found at Element nodes / Element Gauss points

(vii) In FEM we are using, the differential equations are satisfied exactly inside an element. TRUE / FALSE
(viii) In FEM we are using, the essential boundary conditions are satisfied exactly on the boundary. TRUE / FALSE
(ix) In FEM we are using, the natural boundary conditions are satisfied exactly on the boundary. TRUE / FALSE
(x) In iso-parametric elements the primary variables and coordinates are approximated by the same interpolation functions. TRUE / FALSE

2. A beam has a uniform load and a moment applied to it as shown. Model the beam using two equal beam elements. Assume EI is a constant for the beam. (a) Write the global stiffness matrix and the right hand side vector *before incorporating the loads and boundary conditions*. In terms of w, L, E and I determine : (b) the slope at B; (c) the reaction force at C; (d) the deflection at x =L/2.



θ_B=--------------------------

R_C =-------------------------------

v(L/2) =-------------------------------

3. (a) Derive the boundary value problem on u by finding the stationary value of the functional I below.

$$I = \iint_{\Omega} F(u, u_x, u_y, x, y)\,dx\,dy + \oint_{\Gamma} g u\,ds \qquad \text{where} \qquad u_x = \frac{\partial u}{\partial x} \qquad u_y = \frac{\partial u}{\partial y}$$

Ω is the domain with the boundary Γ, and g is a function defined on the boundary.
(b) Apply your results of part (a) to obtain the specific boundary value problem for the functional below.

$$I = \iint_{\Omega} [2xu_x^2 + 3u_x u_y + y^2 u_y^2 - 5u]\,dx\,dy + \oint_{\Gamma} \sin\left(\frac{\pi x}{a}\right) u\,ds$$

(b) If the geometry is the rectangle shown, write the boundary conditions at x = a and y =b.