Final MEEM 4150 Dec. 16th, 2003

1. A thin beam cross-section of uniform thickness t is shown in Fig. 1. Determine the shear center location with respect to point A. Assume $t \ll a$.

Fig. 1

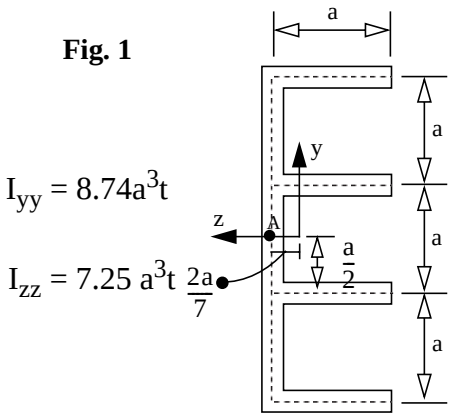
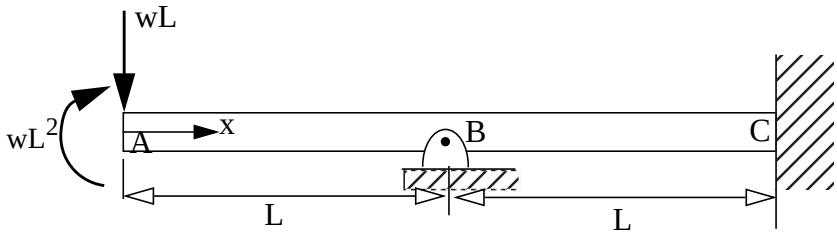


Fig. 2



2. Using energy methods find the reaction force at B and the slope of the beam shown in Fig. 2 at B in terms of E, I, w, and L.

3. The steel (G = 80 GPa, τ_{yield} = 160 MPa) shaft shown has a plastic zone 30 mm deep in section AB.
   (a) Determine the magnitude of the applied torque T_{ext}.
   (b) The rotation of section C with respect to the wall at A.

Fig. 3

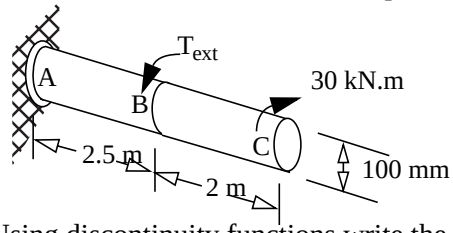
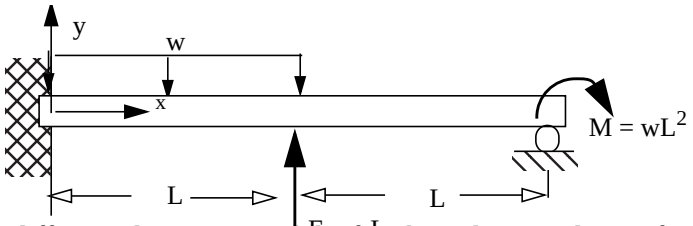


Fig. 4



4. (a) Using discontinuity functions write the *fourth order* differential equation and the four boundary conditions for the beam and loading shown in Fig. 4. DO NOT INTEGRATE or SOLVE.

(b) On a point in steel (E = 30,000 ksi ; ν = 0.25 ; α = 6.6 μ /^{o}F) measurements show an increase in temperature of 100^{o}F and the stresses as $\sigma_{xx} = 12\ ksi\ (T)$ $\sigma_{yy} = 16\ ksi\ (T)$ $\tau_{xy} = -10\ ksi$. Determine the normal strain ε_{xx}, assuming plane *strain.*

(c, d) The principal stresses at a point were found to be σ_1= 40 ksi (T), σ_2= 10 ksi (T), σ_3= 20 ksi (C). Using these stress values solve parts c and d

(c) Determine the second stress invariant at the point.

(d) The critical stress intensity factor for the material is $22 ksi\sqrt{in}$, what would be the critical crack length at that point.

(e) The displacements in a body are given by:

$$u = [0.5(x^2 - y^2) + 0.5xy](10^{-3})mm \qquad v = [0.25(x^2 - y^2) - xy](10^{-3})mm$$

Determine the strain γ_{xy} at x = 5 mm and y = 7 mm.

ANSWERS

1. $e_y = 0$ $e_z = 0.3448a$ 2. $R_B = wL$ $\left(\frac{dv}{dx}\right)_B = 0$

3. T_{ext} = 71.22 kN-m $\phi_c - \phi_A$=0.1736 rads CW

4 b. ε_{xx} = 1033.3 μ (c) I2 = -600 ksi^2 (d) Crack length = 0.193 in (e) γ_{xy} = -9000 μ