1. Using energy methods find the reaction force at A and the slope of the beam at A in terms of $E$, $I$, $w$, and $L$.

2. A thin walled open cross-section with a uniform thickness ‘t’ is shown. Assume that $t \ll a$, i.e., terms of $t$, $t^2$ and $t^3$ can be neglected in comparison to ‘a’. Determine the coordinates of the shear center $e_y$ and $e_z$ with respect to the centroid at C.

3. An aluminum rod is securely fastened to a rigid plate that does not rotate during deformation. A gap of 0.5 mm exists between the rigid plate and the steel rod in the undeformed state. The properties of the two rods are given in Table 1. In addition to the applied load the temperature of both rods is raised by 100°. (a) Determine the movement of the rigid plate (b) The axial stress in steel.

<table>
<thead>
<tr>
<th></th>
<th>diameter (mm)</th>
<th>$E$ (GPa)</th>
<th>$\alpha$ (μ/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>20</td>
<td>70</td>
<td>22.5</td>
</tr>
<tr>
<td>Steel</td>
<td>10</td>
<td>210</td>
<td>12</td>
</tr>
</tbody>
</table>

4. (a) Using Discontinuity Functions write the fourth order differential equation and the four boundary conditions. DO NOT INTEGRATE or SOLVE.

(b) The displacement of the beam in the y-direction is as given. If the bending rigidity (EI) is $135 \times 10^6$ lbs-in²
determine the moment \( M_B \) and reaction force \( R_B \).

\[
\begin{align*}
v_1 &= 5(x^3 - 20x^2) \times 10^{-6} \text{ in} \quad 0 \leq x \leq 20 \\
v_2 &= 5(x^3 - 800x + 8000) \times 10^{-6} \text{ in} \quad 20 \leq x \leq 40
\end{align*}
\]

(c) For a non-linear material, the bending normal stress across the cross-section was found to be

\[
\sigma_{xx} = \begin{cases} 
-K(-y)^{0.75} & y \leq 0 \\
K(y)^{0.75} & y \geq 0
\end{cases}
\]

Determine the equivalent internal bending moment in terms.

In parts (d) and (e) below use the composite cross-section shown in which \( E_1 = 10,000 \text{ ksi} \) and \( E_2 = 30,000 \text{ ksi} \). Point A is at the bottom of the cross-section and points B and C are at the top of the cross-section.

(d) The AXIAL normal stress at A is 12 ksi (T). What are the axial stresses at points B and C?

\[
\sigma_B = \quad \sigma_C =
\]

(e) The BENDING normal stress at A is 12 ksi (T). What are the bending normal stresses at B and C?

\[
\sigma_B = \quad \sigma_C =
\]

ANSWERS

1. \( R_A = 7wL/64 \)
2. \( e_y = 0.5778a \)
3. \( \delta_{\text{plate}} = 1.58 \text{ mm} \)
4(b) \( M_B = 27,000 \text{ in-lb} \)
5. \( e_z = 0 \)
6. \( \sigma_{\text{steel}} = 441 \text{ MPa} \)
7. \( R_B = 0 \)
8. \( M_z = -0.727Kta^{2.75} \)

(d) \( \sigma_B = 12 \text{ ksi(T)} \) \( \sigma_C = 4 \text{ ksi(T)} \)

(e) \( \sigma_B = 12 \text{ ksi(C)} \) \( \sigma_C = 4 \text{ ksi(C)} \)