1. (a) Determine $I_{yz}$ for the cross-section shown.

$$I_{yz} = \frac{6 \text{ in} \times 1 \text{ in} \times 1 \text{ in} \times 1 \text{ in}}{6 \text{ in}}$$

(b) The internal bending moments on a cross-section shown were determined to be $M_y = -20 \text{ kN m}$ and $M_z = (25 \text{ kN m}).$ Determine the orientation of the neutral axis and show it on the cross-section.

(c) The normal strain at point A on a cross-section of a laminated beam in bending was found to be $-1000 \mu.$ Sketch the normal stress distribution along the line parallel to the y-axis and label the stress values. Use $E_{\text{steel}} = 30,000 \text{ ksi}$ and $E_{\text{wood}} = 8,000 \text{ ksi}.$

(d) A beam of elastic-perfectly plastic material has a yield stress of 30 ksi. Point A on the cross-section shown just reaches yield stress in compression at a given load. (i) Sketch the stress distribution along a line parallel to the y-axis. (ii) Write the expressions for bending normal stress $\sigma_{xx}$ and the intervals over which each expression is valid. Use coordinate $y$ and parameter ‘a’ representing the distance of elastic-plastic boundary from neutral axis in writing your stress expressions.
(e) A thin cross-section of uniform thickness $t$ is shown below. If shear stresses were to be found at point A then what values of $Q_y$ and $Q_z$ are needed for the calculation. Assume $t \ll a$ and gap at D is of negligible thickness. Report the values of $Q_y$ and $Q_z$ in terms of $t$ and $a$.

$$Q_y = \frac{-0.75a^2t}{2a^2t}$$

$$Q_z = \frac{2a^2t}{2a^2t}$$

(f) A circular steel ($E= 30,000$ ksi, $v=0.29$, $\alpha = 6.6 \mu^0F$) bar is held between two rigid walls. Determine the change in diameter of the bar if temperature of the bar is raised by $100^\circ$F.

$$\Delta d = \frac{2.55 \times 10^{-3}}{10 \times 10^{-3}} \text{ in.}$$

2. (a) A cross-section has a uniform thickness $t$. Assuming $t$ is small determine the shear center with respect to point C in terms of $h$.

$$I_{yy} = 5.33h^3t$$

$$I_{xz} = 9.33h^3t$$

$$I_{yz} = 0$$

$$e_y = \frac{-1.714h}{2h}$$

$$e_z = \frac{-2h}{2h}$$

3. A shaft made from elastic-perfectly plastic has a shear yield stress of $15$ ksi and a shear modulus of $12,000$ ksi. The plastic zone in shaft segment AB was found to be $0.25$ inch deep. (a) Determine the external torque $T_{ext}$. (b) The rotation of section at B.

$$T_{ext} = \frac{158.8 \text{ in-kip}}{70 \text{ in}}$$

$$\phi_B = -0.05 \text{ rad}$$

ANSWERS

1(a) $I_{yz} = -15.75 \text{ in}^4$  (b) $\theta_{N,A} = 65.04^\circ$ from z 

(d) $\sigma_{xx} = \begin{cases} 
30\text{ksi} & -(11 - a) \leq y \leq -a \\
-30\frac{y}{a} & -a \leq y \leq (a - 2) \\
-30\frac{y}{a} & (a - 2) \leq y \leq a 
\end{cases}$

(e) $Q_y = -0.75a^2t$  $Q_z = 2a^2t$  

$\Delta d = 2.55(10^{-3}) \text{ in.}$

2. $e_y = 0$  $e_z = 1.714h$

3. $T_{ext} = 158.8 \text{ in-kip}$  $\phi_B = -0.05 \text{ rad}$